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Computer Program for Numerical Evaluation of the Performance of a TM_{01} Circular to TE_{10} Rectangular Waveguide Mode Converter

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Two identical rectangular waveguides are fed by means of two symmetrically placed apertures in a circular waveguide. Power is transmitted from the TM_{01} mode in the circular waveguide to the TE_{10} dominant mode in each rectangular waveguide. A computer program for evaluating this power is described and listed. This computer program also evaluates the magnitude of each orthogonal component of the tangential electric field in the apertures. Sample input and output are provided. Several plots of data obtained by means of the computer program appear in a previous technical report.

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Contents

Chapter 1

Introduction

Chapter 1 contains a statement of the waveguide mode converter problem and a description of the general method of solution. Chapter 1 is self-contained; it can be read without supplementary references. A computer program was written to obtain a numerical solution of the waveguide mode converter problem. Chapter 2 contains all the information necessary to run this program and to interpret the final numerical results that it writes out. Chapter 2 is more or less self-contained. References [1] and [2] are cited in Chapter 2. Although these references are enlightening, one can grasp the contents of Chapter 2 without delving into them.

The computer program consists of a main program, the subroutines MODES, BESIN, BES, INTERPOL, PHI, and DGN, the function subprogram FXY, and the subroutines DECOMP and SOLVE in that order. These parts of the program are described and listed in Chapters 3 to 11. Unfortunately, Chapters 3 to 11 are not self-contained because they depend heavily on equations extracted from [2]. One must read both [1] and [2] in order to verify these equations. Considerable time and effort are required to read both [1] and [2]. Fortunately, one who wants to use the computer program does not have to read Chapters 3 to 11 unless unforeseen difficulty arises while running the program. If one experiences such difficulty or if one desires to modify the program to suit one's needs, then one will have to go into Chapters 3 to 11. Most often, one will not have to read all of Chapters 3 to 11, and one will be able to use equations extracted from [2] without going through their derivations in [1] and [2].

1.1 Statement of the Problem

There is, as shown in Fig. 1, a circular waveguide which is closed at one end. Two symmetrically placed apertures in the lateral wall of this waveguide are backed by rectangular waveguides of identical dimensions. The interiors of the left-hand rectangular waveguide, the right-hand rectangular waveguide and the circular waveguide are called regions 1, 2, and 3, respectively. Homogeneous space of permeability μ and permittivity ϵ exists in all of these regions. The excitation is a TM_{01} wave of unit amplitude traveling in the z-direction in the circular waveguide. The circular waveguide is of radius a and is terminated by a perfectly conducting wall at $z = L_3$. The radius a is such that only the TE_{11} and TM_{01} modes can propagate in the circular waveguide.

Both rectangular waveguides run parallel to the x-axis. Both have the the same cross section $(-\frac{b}{2} \leq y \leq \frac{b}{2},\ -\frac{c}{2} \leq z \leq \frac{c}{2})$ where $c < b$ and b is such that only the TE_{10} dominant mode can propagate in each rectangular waveguide. The aperture A_1 which feeds the left-hand rectangular waveguide in Fig. 1 is the surface for which $(\rho = a,\ \pi - \phi_o \leq \phi \leq \pi + \phi_o,\ -\frac{c}{2} \leq z \leq \frac{c}{2})$ where ρ and ϕ are the cylindrical coordinates related to x and y by

$$\rho = \sqrt{x^2 + y^2} \tag{1.1}$$

$$\tan\phi = \frac{y}{x} \tag{1.2}$$

and

$$\phi_o = \sin^{-1}\left(\frac{b}{2a}\right). \tag{1.3}$$

The aperture A_2 which feeds the right-hand rectangular waveguide is the surface for which $(\rho = a,\ -\phi_o \leq \phi \leq \phi_o,\ -\frac{c}{2} \leq z \leq \frac{c}{2})$.

The voltage to current ratio of the TE_{10} mode at $x = -L_1$ in region 1 is taken to be Z_1. All other rectangular waveguide modes are evanescent. The voltage to current ratios of the evanescent modes at $x = -L_1$ do not come into play because L_1 is taken to be so large that any evanescent wave emanating from the termination at $x = -L_1$ will have negligibly small amplitude upon arrival at aperture A_1. The voltage to current ratio of the TE_{10} mode at $x = L_2$ in region 2 is taken to be Z_2. Here, L_2 is taken to be so large that

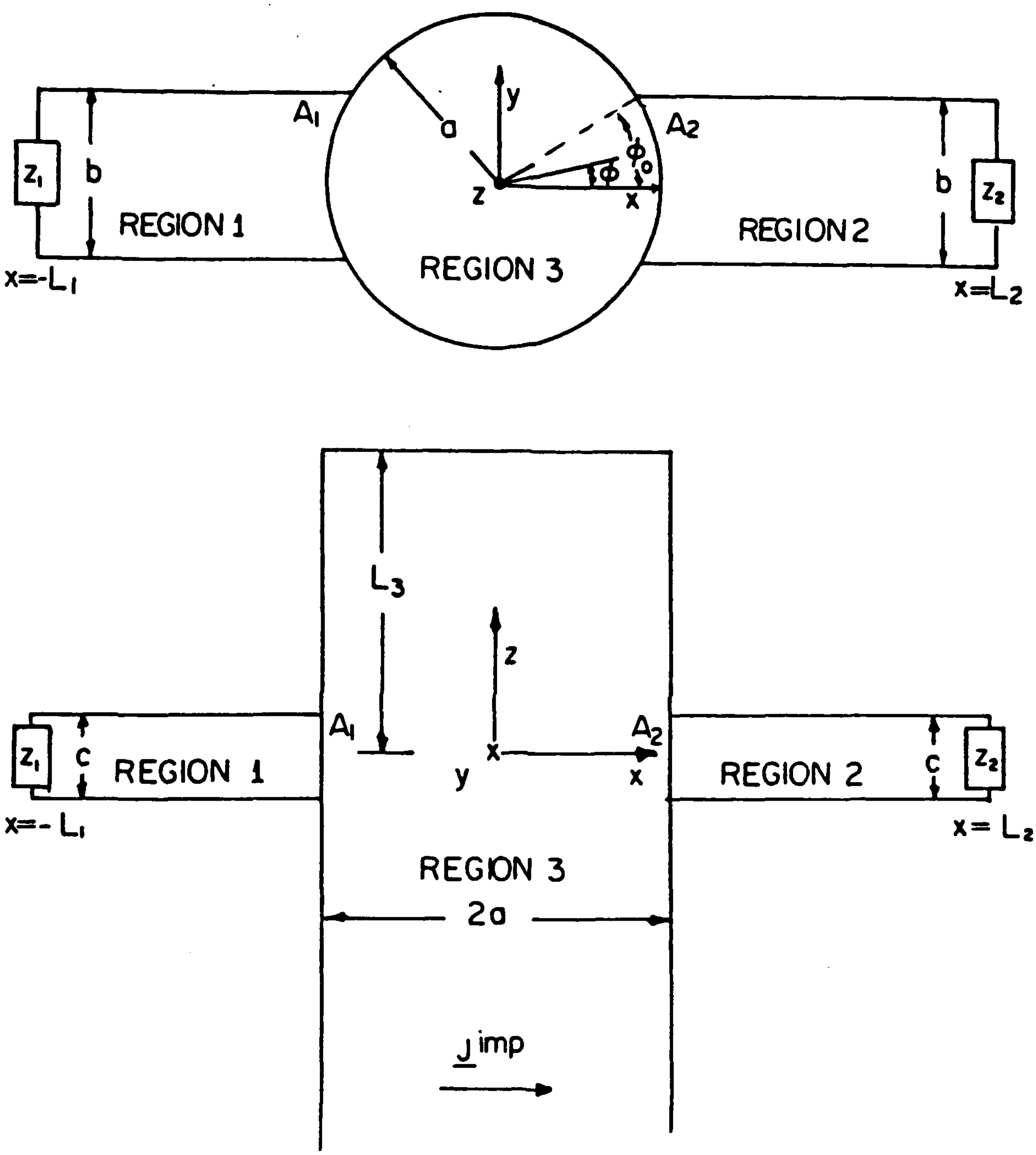

Fig. 1. Top and side views of the TM_{01} to TE_{10} mode converter.

any evanescent wave emanating from the termination at $x = L_2$ will have negligibly small amplitude upon arrival at aperture A_2.

As previously stated, the excitation is a z-traveling (traveling in the z-direction) TM_{01} wave of unit amplitude in the circular waveguide. By assumption, this is the only z-traveling wave at $z = -c/2$. Concerning the fields in the part of region 3 for which $z > -c/2$ and in all of regions 1 and 2, it does not matter how this z-traveling TM_{01} wave is produced. We let this wave be produced by a sheet of electric current $\underline{J}^{\text{imp}}$ located at $z < -c/2$ in the circular waveguide. For simplicity, we assume that the circular waveguide is terminated at $z << -c/2$ by a matched load, i.e., any $-z$-traveling wave in the region for which $z < -c/2$ is never reflected. The problem is to find out how much power of the z-traveling TM_{01} wave is transmitted into the rectangular waveguides and to calculate the magnitudes of the ϕ- and z-components of the electric field in the apertures A_1 and A_2.

1.2 Method of Solution

Seeking to solve the above-mentioned problem by means of the generalized network formulation for aperture problems [3], we close the apertures A_1 and A_2 with perfect conductors of infinitesimal thickness. As shown in Fig. 2, we place the surface density of magnetic current $\underline{M}^{(1)}$ on the region 1 side of the closed aperture A_1, $-\underline{M}^{(1)}$ on the region 3 side of A_1, $\underline{M}^{(2)}$ on the region 2 side of the closed aperture A_2, and $-\underline{M}^{(2)}$ on the region 3 side of A_2. The magnetic currents in Fig. 2 are supposed to be located right on (infinitesimal distances from either side) the closing conductors. The finite displacement of these magnetic currents from the closing conductors in Fig. 2 is only for the purpose of illustration. With the arrangement of magnetic currents in Fig. 2, the tangential electric field is continuous across A_1 and A_2. Now, the fields in Fig. 2 will be the same as those in Fig. 1 if $\underline{M}_1^{(1)}$ and $\underline{M}_2^{(2)}$ are adjusted such that the tangential magnetic field is continuous across A_1 and A_2.

Continuity of the tangential magnetic field across A_1 is expressed as

$$-\underline{H}^{(1)}_{\tan}(\underline{0}, \underline{M}^{(1)}) - \underline{H}^{(3)}_{\tan}(\underline{0}, \underline{M}^{(1)}) - \underline{H}^{(3)}_{\tan}(\underline{0}, \underline{M}^{(2)}) = -\underline{H}^{(3)}_{\tan}(\underline{J}^{\text{imp}}, \underline{0}) \quad (1.4)$$

where $\underline{H}^{(r)}_{\tan}(\underline{J}, \underline{M})$ is the tangential magnetic field radiated by the combination of the electric current $\underline{J}$ and the magnetic current $\underline{M}$ in region r where

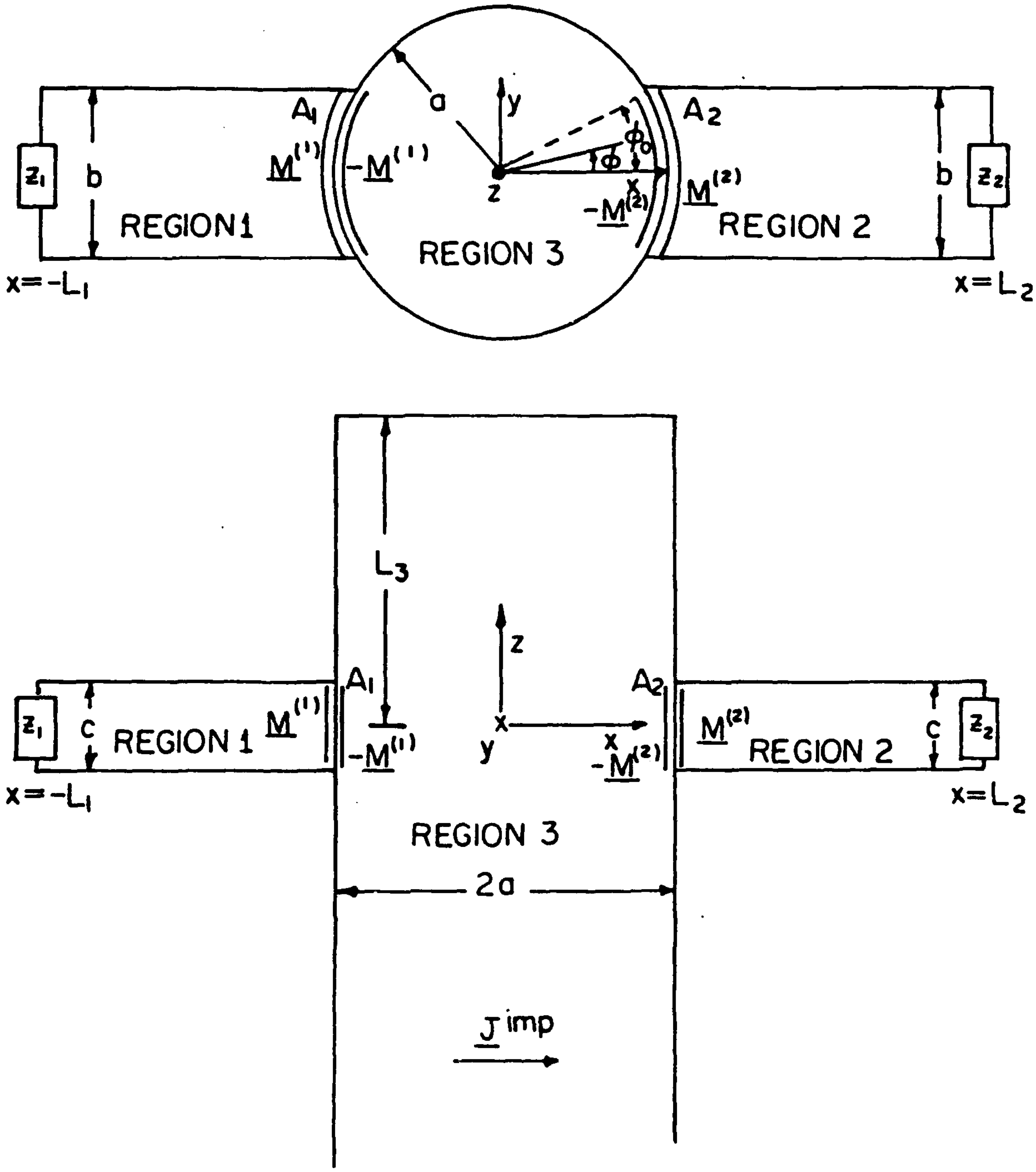

Fig. 2. Top and side views of the situation equivalent to that of Fig. 1.

$r = 1$ or 3. In (1.5) to follow, $r = 2$ or 3. Continuity of the tangential magnetic field across A_2 is expressed as

$$-\underline{H}^{(3)}_{\tan}(\underline{0}, \underline{M}^{(1)}) - \underline{H}^{(2)}_{\tan}(\underline{0}, \underline{M}^{(2)}) - \underline{H}^{(3)}_{\tan}(\underline{0}, \underline{M}^{(2)}) = -\underline{H}^{(3)}_{\tan}(\underline{J}^{\text{imp}}, \underline{0}). \quad (1.5)$$

Equation (1.4) is supposed to be valid on A_1, and (1.5) is supposed to be valid on A_2.

1.2.1 Expansion Functions

Seeking to solve (1.4) and (1.5) for $\underline{M}^{(1)}$ and $\underline{M}^{(2)}$ by the method of moments [4], we let

$$\underline{M}^{(1)} = \sum_{q=1}\sum_{p=1} V^{1\text{TM}}_{pq} \underline{M}^{1\text{TM}}_{pq}(\phi, z) + \sum_{q=0} \sum_{\substack{p=0 \\ p+q\neq 0}} V^{1\text{TE}}_{pq} \underline{M}^{1\text{TE}}_{pq}(\phi, z) \quad (1.6)$$

$$\underline{M}^{(2)} = \sum_{q=1}\sum_{p=1} V^{2\text{TM}}_{pq} \underline{M}^{2\text{TM}}_{pq}(\phi, z) + \sum_{q=0} \sum_{\substack{p=0 \\ p+q\neq 0}} V^{2\text{TE}}_{pq} \underline{M}^{2\text{TE}}_{pq}(\phi, z). \quad (1.7)$$

Upper limits of the summation indices in (1.6) and (1.7) will be chosen later. In (1.6) and (1.7), the V's are unknown coefficients to be determined, and $\underline{M}^{1\delta}_{pq}(\phi, z)$ and $\underline{M}^{2\delta}_{pq}(\phi, z)$ are expansion functions given by

$$\underline{M}^{1\delta}_{pq}(\phi, z) = \underline{u}_\phi e^{\delta}_{zpq}(y^{1+}, z^+) + \underline{u}_z \frac{\sin\phi_o}{\phi_o} e^{\delta}_{ypq}(y^{1+}, z^+),$$
$$\left\{ \begin{array}{c} \rho = a \\ \pi - \phi_o \le \phi \le \pi + \phi_o \\ -\frac{c}{2} \le z \le \frac{c}{2} \end{array} \right. \quad (1.8)$$

$$\underline{M}^{2\delta}_{pq}(\phi, z) = \underline{u}_\phi e^{\delta}_{zpq}(y^{2+}, z^+) - \underline{u}_z \frac{\sin\phi_o}{\phi_o} e^{\delta}_{ypq}(y^{2+}, z^+), \left\{ \begin{array}{c} \rho = a \\ -\phi_o \le \phi \le \phi_o \\ -\frac{c}{2} \le z \le \frac{c}{2} \end{array} \right. \quad (1.9)$$

where δ is either TM or TE. Moreover, $\underline{u}_\phi$ and $\underline{u}_z$ are the unit vectors in the ϕ- and z-directions, respectively. Furthermore,

$$y^{1+} = (\pi - \phi)x_o + \tfrac{b}{2} \tag{1.10}$$

$$y^{2+} = \phi x_o + \tfrac{b}{2} \tag{1.11}$$

$$z^{+} = z + \tfrac{c}{2} \tag{1.12}$$

$$x_o = \frac{a \sin \phi_o}{\phi_o}. \tag{1.13}$$

In (1.8) and (1.9), e_{ypq}^{TM} and e_{zpq}^{TM} are the y- and z-components of the TM rectangular waveguide mode function $\underline{e}_{pq}^{\mathrm{TM}}$[†] given by eq. (A.10)[‡] of [1]:

$$e_{ypq}^{\mathrm{TM}}(y^+, z^+) = -\frac{2\pi}{k_{pq}\sqrt{bc}}\,\frac{p}{b}\cos\left(\frac{p\pi y^+}{b}\right)\sin\left(\frac{q\pi z^+}{c}\right) \tag{1.14}$$

$$e_{zpq}^{\mathrm{TM}}(y^+, z^+) = -\frac{2\pi}{k_{pq}\sqrt{bc}}\,\frac{q}{c}\sin\left(\frac{p\pi y^+}{b}\right)\cos\left(\frac{q\pi z^+}{c}\right), \tag{1.15}$$

and e_{ypq}^{TE} and e_{zpq}^{TE} are the y- and z-components of the TE rectangular waveguide mode function $\underline{e}_{pq}^{\mathrm{TE}}$[§] given by eq. (A.23) of [1]:

$$e_{ypq}^{\mathrm{TE}}(y^+, z^+) = \frac{\pi}{k_{pq}}\sqrt{\frac{\epsilon_p\epsilon_q}{bc}}\,\frac{q}{c}\cos\left(\frac{p\pi y^+}{b}\right)\sin\left(\frac{q\pi z^+}{c}\right) \tag{1.16}$$

$$e_{zpq}^{\mathrm{TE}}(y^+, z^+) = -\frac{\pi}{k_{pq}}\sqrt{\frac{\epsilon_p\epsilon_q}{bc}}\,\frac{p}{b}\sin\left(\frac{p\pi y^+}{b}\right)\cos\left(\frac{q\pi z^+}{c}\right). \tag{1.17}$$

[†] The transverse parts of the modal electric fields $\underline{E}_{pq}^{\mathrm{TM}+}$ of eq. (A.2) of [1] and $\underline{E}_{pq}^{\mathrm{TM}-}$ of eq. (A.3) of [1] are proportional to $\underline{e}_{pq}^{\mathrm{TM}}$.

[‡] An equation that appears in a reference will be cited by placing "eq." before the equation number. An equation that appears in the present report will be cited by writing only its number in parentheses. However, an equation number at the beginning of a sentence will always be preceded by "Equation".

[§] The modal electric fields $\underline{E}_{pq}^{\mathrm{TE}+}$ of eq. (A.14) of [1] and $\underline{E}_{pq}^{\mathrm{TE}-}$ of eq. (A.15) of [1] are equal to $\underline{e}_{pq}^{\mathrm{TE}}$.

1.2.2 Approximations to the Expansion Functions Radiate Rectangular Waveguide Modes in Regions 1 and 2

The expansion functions $\underline{M}_{pq}^{1\delta}(\phi, z)$ and $\underline{M}_{pq}^{2\delta}(\phi, z)$ of (1.8) and (1.9) were chosen such that

$$\underline{M}_{pq}^{1\delta}(\phi, z) \approx -\underline{M}_{pq}^{\delta}(y^{1+}, z^{+}), \begin{cases} x = -x_o \\ 0 \leq y^{1+} \leq b \\ 0 \leq z^{+} \leq c \end{cases} \tag{1.18}$$

$$\underline{M}_{pq}^{2\delta}(\phi, z) \approx \underline{M}_{pq}^{\delta}(y^{2+}, z^{+}), \begin{cases} x = x_o \\ 0 \leq y^{2+} \leq b \\ 0 \leq z^{+} \leq c \end{cases} \tag{1.19}$$

where y^{1+}, y^{2+}, z^{+}, and x_o are given by (1.10)–(1.13) and

$$\underline{M}_{pq}^{\delta}(y^{+}, z^{+}) = \underline{u}_y e_{zpq}^{\delta}(y^{+}, z^{+}) - \underline{u}_z e_{ypq}^{\delta}(y^{+}, z^{+}). \tag{1.20}$$

If ϕ_o is small, the plane surface in (1.18) is a good approximation to the curved surface in (1.8) because $-x_o$ is the average value of x on the curved surface in (1.8). If ϕ_o is small, the plane surface in (1.19) is a good approximation to the curved surface in (1.9) because x_o is the average value of x on the curved surface in (1.9).

The magnetic current $-\underline{M}_{pq}^{\delta}(y^{1+}, z^{+})$ on the right-hand side of (1.18) gives rise to a voltage dv^a at $(y^{1+} + dy^{1+}, z^{+} + dz^{+})$ with respect to the voltage at (y^{1+}, z^{+}). The superscript "a" in "dv^a" stands for approximate. Here, dv^a is measured on the side of $-\underline{M}_{pq}^{\delta}(y^{1+}, z^{+})$ facing region 1. Now,

$$dv^a = -\left\{\underline{u}_x \times \underline{M}_{pq}^{\delta}(y^{1+}, z^{+})\right\} \cdot (\underline{u}_y \, dy^{1+} + \underline{u}_z \, dz^{+}). \tag{1.21}$$

The magnetic current $\underline{M}_{pq}^{1\delta}(\phi, z)$ on the left-hand side of (1.18) gives rise to a voltage dv at $(\phi + d\phi, z + dz)$ with respect to the voltage at (ϕ, z). Here, dv is measured on the side of $\underline{M}_{pq}^{1\delta}(\phi, z)$ facing region 1. Now,

$$dv = -\left\{\underline{u}_\rho \times \underline{M}_{pq}^{1\delta}(\phi, z)\right\} \cdot (\underline{u}_\phi a \, d\phi + \underline{u}_z \, dz). \tag{1.22}$$

The approximation (1.18) will be good if ϕ_o is small and if

$$dv^a = dv. \tag{1.23}$$

Substitution of (1.8) into (1.22) leads to

$$dv = e^{\delta}_{ypq}(y^{1+}, z^{+})x_o\, d\phi - e^{\delta}_{zpq}(y^{1+}, z^{+})\, dz. \tag{1.24}$$

Substitution of (1.20) into (1.21) leads to

$$dv^a = -e^{\delta}_{ypq}(y^{1+}, z^{+})\, dy^{1+} - e^{\delta}_{zpq}(y^{1+}, z^{+})\, dz^{+}. \tag{1.25}$$

From (1.10) and (1.12),

$$dy^{1+} = -x_o\, d\phi \tag{1.26}$$

$$dz^{+} = dz. \tag{1.27}$$

Substitution of (1.26) and (1.27) into (1.25) gives

$$dv^a = e^{\delta}_{ypq}(y^{1+}, z^{+})x_o\, d\phi - e^{\delta}_{zpq}(y^{1+}, z^{+})\, dz. \tag{1.28}$$

Since the right-hand side of (1.28) is equal to that of (1.24), (1.23) is satisfied. Therefore, the approximation (1.18) will be good if ϕ_o is small. Similarly, it can be shown that the approximation (1.19) will be good if ϕ_o is small.

One can verify that

$$-\underline{M}^{\delta}_{pq}(y^{1+}, z^{+}) = -\underline{e}^{\delta}_{pq}(y^{1+}, z^{+}) \times \underline{u}_x. \tag{1.29}$$

Therefore, when placed on the region 1 side of a conductor that closes the approximate aperture surface described on the right-hand side of (1.18), the magnetic current $-\underline{M}^{\delta}_{pq}(y^{1+}, z^{+})$ on the right-hand side of (1.18) produces an electric field whose transverse part is $\underline{e}^{\delta}_{pq}(y^{1+}, z^{+})$ on the region 1 side of this magnetic current. Hence, the magnetic current on the right-hand side of (1.18) excites only the pq^{th} δ† rectangular waveguide mode in region 1. One can verify that

$$\underline{M}^{\delta}_{pq}(y^{2+}, z^{+}) = \underline{e}^{\delta}_{pq}(y^{2+}, z^{+}) \times \underline{u}_x. \tag{1.30}$$

Therefore, when placed on the region 2 side of a conductor that closes the approximate aperture surface described on the right-hand side of (1.19), the magnetic current $\underline{M}^{\delta}_{pq}(y^{2+}, z^{+})$ on the right-hand side of (1.19) produces an electric field whose transverse part is $\underline{e}^{\delta}_{pq}(y^{2+}, z^{+})$ on the region 2 side of this magnetic current. Hence, the magnetic current on the right-hand side of (1.19) excites only the pq^{th} δ rectangular waveguide mode in region 2.

†Recall that δ is either TM or TE.

1.2.3 The Matrix Equation

The symmetric product $< A, B >$ of two vectors $\underline{A}$ and $\underline{B}$ is, by definition, the surface integral of their dot product over whichever aperture they are defined:

$$< A, B >= \iint_{A_1 \text{ or } A_2} \underline{A} \cdot \underline{B}\, ds. \tag{1.31}$$

Here, ds is the differential element of surface area. Substituting (1.6) and (1.7) into (1.4) and taking the symmetric product of (1.4) with each of the expansion functions $\{\underline{M}_{mn}^{1\mathrm{TM}}\}$ and $\{\underline{M}_{mn}^{1\mathrm{TE}}\}$ and then substituting (1.6) and (1.7) into (1.5) and taking the symmetric product of (1.5) with each of the expansion functions $\{\underline{M}_{mn}^{2\mathrm{TM}}\}$ and $\{\underline{M}_{mn}^{2\mathrm{TE}}\}$, we obtain the matrix equation

$$[Y^1 + Y^2 + Y^3] \begin{bmatrix} \vec{V}^{1\mathrm{TM}} \\ \vec{V}^{1\mathrm{TE}} \\ \vec{V}^{2\mathrm{TM}} \\ \vec{V}^{2\mathrm{TE}} \end{bmatrix} = \begin{bmatrix} \vec{I}^{1\mathrm{TM}} \\ \vec{I}^{1\mathrm{TE}} \\ \vec{I}^{2\mathrm{TM}} \\ \vec{I}^{2\mathrm{TE}} \end{bmatrix} \tag{1.32}$$

where the Y's are square matrices and the $\vec{V}$'s and the $\vec{I}$'s are column vectors. The j^{th} element of $\vec{V}^{\gamma\delta}$ is $V_j^{\gamma\delta}$ given by

$$V_j^{\gamma\delta} = V_{pq}^{\gamma\delta}, \begin{cases} j = 1, 2, \cdots, N^\delta \\ \gamma = 1, 2 \\ \delta = \mathrm{TM}, \mathrm{TE} \end{cases} \tag{1.33}$$

where N^δ is the maximum value of j. The subscript j is a condensation of the double subscript pq; one and only one positive integer j[†] is assigned to each combination of subscripts p and q. The condensation of pq into j depends on δ. In the first double summation on the right-hand side of (1.6) where δ is TM, p and q run though all positive integers such that

$$\sqrt{(p\pi)^2 + \left(\frac{q\pi b}{c}\right)^2} \leq \mathrm{BKM} \tag{1.34}$$

where BKM enters as input data into the computer program that will be described and listed in subsequent chapters. In the second double summation on the right-hand side of (1.6) where δ is TE, p and q run through all

† $j = 1, 2, \cdots$.

nonnegative integers except $p = q = 0$ such that (1.34) is satisfied. The i^{th} element of $\vec{I}^{\alpha\beta}$ is $I_i^{\alpha\beta}$ given by

$$I_i^{\alpha\beta} = -\iint_{A_\alpha} \underline{M}_{mn}^{\alpha\beta} \cdot \underline{H}^{(3)}(\underline{J}^{\text{imp}}, \underline{0})\, ds, \quad \begin{cases} i = 1, 2, \cdots, N^\beta \\ \alpha = 1, 2 \\ \beta = \text{TM,TE} \end{cases} \tag{1.35}$$

where N^β is the maximum value of i. If $\beta = \delta$, i would be the same condensation of mn that j was of pq in (1.33). The subscript "tan" need not be affixed to $H^{(3)}$ in (1.35) because $\underline{M}_{mn}^{\alpha\beta}$ is tangent to A_α. In (1.32),

$$Y^1 = \begin{bmatrix} Y^{1,1\text{TM},1\text{TM}} & Y^{1,1\text{TM},1\text{TE}} & 0 & 0 \\ Y^{1,1\text{TE},1\text{TM}} & Y^{1,1\text{TE},1\text{TE}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{1.36}$$

$$Y^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & Y^{2,2\text{TM},2\text{TM}} & Y^{2,2\text{TM},2\text{TE}} \\ 0 & 0 & Y^{2,2\text{TE},2\text{TM}} & Y^{2,2\text{TE},2\text{TE}} \end{bmatrix} \tag{1.37}$$

$$Y^3 = \begin{bmatrix} Y^{3,1\text{TM},1\text{TM}} & Y^{3,1\text{TM},1\text{TE}} & Y^{3,1\text{TM},2\text{TM}} & Y^{3,1\text{TM},2\text{TE}} \\ Y^{3,1\text{TE},1\text{TM}} & Y^{3,1\text{TE},1\text{TE}} & Y^{3,1\text{TE},2\text{TM}} & Y^{3,1\text{TE},2\text{TE}} \\ Y^{3,2\text{TM},1\text{TM}} & Y^{3,2\text{TM},1\text{TE}} & Y^{3,2\text{TM},2\text{TM}} & Y^{3,2\text{TM},2\text{TE}} \\ Y^{3,2\text{TE},1\text{TM}} & Y^{3,2\text{TE},1\text{TE}} & Y^{3,2\text{TE},2\text{TM}} & Y^{3,2\text{TE},2\text{TE}} \end{bmatrix}. \tag{1.38}$$

In (1.36), the ij^{th} element of the submatrix $Y^{1,1\beta,1\delta}$ is $Y_{ij}^{1,1\beta,1\delta}$ given by

$$Y_{ij}^{1,1\beta,1\delta} = -\int_{A_1} \underline{M}_{mn}^{1\beta} \cdot \underline{H}^{(1)}(\underline{0}, \underline{M}_{pq}^{1\delta})\, ds, \quad \begin{cases} i = 1, 2, \cdots, N^\beta \\ j = 1, 2, \cdots, N^\delta \\ \beta = \text{TM,TE} \\ \delta = \text{TM,TE.} \end{cases} \tag{1.39}$$

In (1.37),

$$Y_{ij}^{2,2\beta,2\delta} = -\int_{A_2} \underline{M}_{mn}^{2\beta} \cdot \underline{H}^{(2)}(\underline{0}, \underline{M}_{pq}^{2\delta})\, ds, \quad \begin{cases} i = 1, 2, \cdots, N^\beta \\ j = 1, 2, \cdots, N^\delta \\ \beta = \text{TM,TE} \\ \delta = \text{TM,TE.} \end{cases} \tag{1.40}$$

In (1.38),

$$Y_{ij}^{3,\alpha\beta,\gamma\delta} = -\int_{A_\alpha} \underline{M}_{mn}^{\alpha\beta} \cdot \underline{H}^{(3)}(\underline{0}, \underline{M}_{pq}^{\gamma\delta})\, ds, \quad \begin{cases} i = 1,2,\cdots,N^\beta \\ j = 1,2,\cdots,N^\delta \\ \alpha = 1,2 \\ \beta = \text{TM,TE} \\ \gamma = 1,2 \\ \delta = \text{TM,TE.} \end{cases} \tag{1.41}$$

In (1.39)–(1.41), j is the same condensation of pq as in (1.33), and i is the same condensation of mn as in (1.35).

In the present report, the Y's and the $\vec{I}$'s in (1.32) are calculated. Then, (1.32) is solved for the $\vec{V}$'s. These $\vec{V}$'s are then used to calculate the following quantities:

1. The time-average power of the $-z$-traveling TM_{01}^{e} wave in the circular waveguide. The superscript "e" indicates that the z-component of the electric field of the wave is even in ϕ.

2. The time-average power of the $-z$-traveling TE_{11}^{e} and TE_{11}^{o} waves in the circular waveguide. The superscript "o" indicates that the z-component of the magnetic field of the wave is odd in ϕ.

3. The time-average powers of the $+z$- and $-z$-traveling TE_{10} waves in the rectangular waveguides.

4. The magnitudes of the ϕ- and z-components of the electric field in the apertures.

If $|E_\phi|$ represents any one of the above quantities, then

$$\begin{aligned} |E_\phi| = |E_\phi^{\text{inc}}| &+ \sum_{q=1}\sum_{p=1} \left(V_{pq}^{1\text{TM}}\, |E_\phi|_{pq}^{1\text{TM}} + V_{pq}^{2\text{TM}}\, |E_\phi|_{pq}^{2\text{TM}} \right) \\ &+ \sum_{q=0} \sum_{\substack{p=0 \\ p+q\neq 0}} \left(V_{pq}^{1\text{TE}}\, |E_\phi|_{pq}^{1\text{TE}} + V_{pq}^{2\text{TE}}\, |E_\phi|_{pq}^{2\text{TE}} \right) \end{aligned} \tag{1.42}$$

where $|E_\phi^{\text{inc}}|$ is what $|E_\phi|$ would be if all the V's were zero. Furthermore, $|E_\phi|_{pq}^{\alpha\beta}$ is what $|E_\phi|$ would be if $\underline{J}^{\text{imp}} = 0$, if $V_{pq}^{\alpha\beta}$ were unity, and if all the other V's were zero. Here, α is 1 or 2, and β is TM or TE. The upper limits of the indices of the summations in (1.42) will be chosen later. These upper limits must be the same as those in (1.6) and (1.7).

Chapter 2

Instructions for Using the Computer Program

The computer program is available on diskette. On diskette, the computer program, which consists of a main program and some subprograms, is stored in the file JAN.92. Sample input data are stored in the file JAN92.DAT. These files are named JAN.92 and JAN92.DAT because January 1992 is the date of the present report, the report in which the computer program is described and listed. The main program, the subroutines MODES, BESIN, BES, INTERPOL, PHI, and DGN, the function subprogram FXY, and the subroutines DECOMP and SOLVE are stored in order in the file JAN.92.

There are two modules of input data in the file JAN92.DAT. The first module of input data is preceded by the three comment statements

```
C      THIS IS THE FIRST MODULE OF INPUT DATA.
C      REMOVE IT FROM THE FILE JAN92.DAT
C      AND PUT IT IN THE FILE IN.DAT.
```

and followed by the comment statement

```
C      THIS IS THE SECOND MODULE OF INPUT DATA.
```

The second module of input data is preceded by the three comment statements

```
C      THIS IS THE SECOND MODULE OF INPUT DATA.
C      REMOVE IT FROM THE FILE JAN92.DAT
C      AND PUT IT IN THE FILE BESIN.DAT.
```

The last line of the second module of input data is the last line in the file JAN92.DAT.

To use the program, first follow the instructions given in the comment statements in the preceding paragraph, i.e., create input data files named IN.DAT and BESIN.DAT, move the first module of input data into IN.DAT, and move the second module of input data into BESIN.DAT. The data in IN.DAT are read by statements in the main program. The data in BESIN.DAT are read by statements in the subroutine BESIN. One can modify the input data to suit his needs. The input data are described in Section 2.1. Modification of the input data may require an increase in the storage area allocated to some arrays. Minimum allocations of arrays are given in Section 2.3. Next, create output data files named OUT.DAT and BESOUT.DAT. Then, give the command or commands necessary to run the program that resides in JAN.92. Running the program causes output data to be written in the output data files OUT.DAT and BESOUT.DAT. The data in OUT.DAT are written by statements in the main program. The data in BESOUT.DAT are written by statements in the subroutine BESIN. The final step in using the program is to interpret the output data. If the program runs without difficulty, only the final output data need be interpreted: intermediate output data can be ignored. The final output data will be described in Section 2.2.1.

The complete computer program, the two modules of input data, and the resulting output data are listed in the present report.† The two modules of input data are listed in Section 2.1.3. The main program, the subroutines MODES, BESIN, BES, INTERPOL, PHI, and DGN, and the function subprogram FXY are listed in Chapters 3 to 10, respectively. The subroutines DECOMP and SOLVE are listed in Chapter 11.

2.1 The Input Data

There are two modules of input data.

†The "resulting output" is the output that is obtained when the computer program is run with the input data listed in the present report.

2.1.1 The First Module of Input Data

The first module of input data is read from the file IN.DAT by means of the following statements early in the main program:

```
      READ(20,10) B,C,L1,L2,L3,BKM,XM,ZL1,ZL2
   10 FORMAT(4D14.7)
      READ(20,144) KAM,BKA0,DBKA,KE3M,NPHI,NZ
  144 FORMAT(I4,2D14.7,3I4)
      READ(20,146)(KE3(I),I=1,KE3M)
  146 FORMAT(15I4)
```

In the first of the above three read statements,

$$\mathrm{B} = \frac{b}{a} \tag{2.1}$$

$$\mathrm{C} = \frac{c}{a} \tag{2.2}$$

$$\mathrm{L1} = \frac{L_1}{a} \tag{2.3}$$

$$\mathrm{L2} = \frac{L_2}{a} \tag{2.4}$$

$$\mathrm{L3} = \frac{L_3}{a}. \tag{2.5}$$

The fields in the rectangular waveguides are expanded as linear combinations of all rectangular waveguide modes whose cutoff wavenumbers do not exceed BKM/b. The cutoff wavenumber of both the TM_{pq} and TE_{pq} rectangular waveguide modes is k_{pq} given by (see eq. (A.8) of [1])

$$k_{pq} = \sqrt{\left(\frac{p\pi}{b}\right)^2 + \left(\frac{q\pi}{c}\right)^2}\,. \tag{2.6}$$

Hence, the values of p and q in (1.6) and (1.7) are limited by (1.34).

The field in the circular waveguide is expanded as a linear combination of a finite number of circular waveguide modes. This number is controlled by the input variable XM. The cutoff wavenumber of both TM^e_{rs} and TM^o_{rs} circular waveguide modes is k^{TM}_{rs} given by (see eqs. (B.7) and (B.30) of [1])

$$k^{\mathrm{TM}}_{rs} = \frac{x_{rs}}{a} \tag{2.7}$$

where x_{rs} is the s^{th} root of

$$J_r(x_{rs}) = 0, \begin{cases} r = 0, 1, 2, \cdots \\ s = 1, 2, 3, \cdots \end{cases} \tag{2.8}$$

where J_r is the cylindrical Bessel function of the first kind of order r. The superscript "e" in TM^e_{rs} indicates that the z-component of the modal electric field is even in ϕ. The superscript "o" in TM^o_{rs} indicates that the z-component of the modal electric field is odd in ϕ. The roots $\{x_{rs},\ s = 1, 2, 3, \cdots\}$ are ordered such that

$$0 < x_{r1} < x_{r2} < x_{r3} \cdots. \tag{2.9}$$

The cutoff wavenumber of both TE^e_{rs} and TE^o_{rs} circular waveguide modes is k^{TE}_{rs} given by (see eqs. (B.41) and (B.59) of [1])

$$k^{TE}_{rs} = \frac{x'_{rs}}{a} \tag{2.10}$$

where x'_{rs} is the s^{th} root of

$$J'_r(x'_{rs}) = 0, \begin{cases} r = 0, 1, 2, \cdots \\ s = 1, 2, 3, \cdots \end{cases} \tag{2.11}$$

where J'_r is the derivative of J_r with respect to its argument. The superscript "e" in TE^e_{rs} indicates that the z-component of the modal magnetic field is even in ϕ. The superscript "o" in TE^o_{rs} indicates that the z-component of the modal magnetic field is odd in ϕ. The roots $\{x'_{rs},\ s = 1, 2, 3, \cdots\}$ are ordered such that

$$0 < x'_{r1} < x'_{r2} < x'_{r3} \cdots. \tag{2.12}$$

The roots $\{x_{rs}\}$ and $\{x'_{rs}\}$ interlace such that (see eq. (B.3) of [2])

$$\left.\begin{array}{l} 0 < x_{01} < x'_{01} < x_{02} < x'_{02} < x_{03} < x'_{03} \cdots \\ r < x'_{r1} < x_{r1} < x'_{r2} < x_{r2} < x'_{r3} < x_{r3} \cdots,\ r = 1, 2, \cdots \end{array}\right\}. \tag{2.13}$$

Truncations of the sequences in (2.13) are

$$\left.\begin{array}{l} 0 < x_{01} < x'_{01} < x_{02} < x'_{02} \cdots x_{0s_{\max}} < x'_{0s_{\max}} \\ r < x'_{r1} < x_{r1} < x'_{r2} < x_{r2} \cdots x'_{rs_{\max}} < x_{rs_{\max}},\ r = 1, 2, \cdots, r_{\max} \end{array}\right\} \tag{2.14}$$

where $s_{\max}$ depends on r. Given r, $s_{\max}$ is the largest positive integer s such that

$$\left.\begin{array}{ll} x_{rs} \leq \text{XM}, & r = 0 \\ x'_{rs} \leq \text{XM}, & r = 1, 2, 3, \cdots, r_{\max} \end{array}\right\} \tag{2.15}$$

where XM is the input variable mentioned prior to (2.7). If (2.15) is not satisfied for $s = 1$, then $s_{\max} = 0$. Assuming that $x_{01} \leq \text{XM}$, $r_{\max}$ is the largest positive integer r such that $x'_{r1} \leq \text{XM}$. The field in the circular waveguide is expanded in terms of all the TM^e_{rs}, TM^o_{rs}, TE^e_{rs}, and TE^o_{rs} circular waveguide modes for which $\{rs\}$ is the full range of subscripts in (2.14).

Instead of using all the circular waveguide modes mentioned in the previous sentence, perhaps we should have used only those TM^e_{rs} and TM^o_{rs} modes for all $\{rs\}$ such that

$$x_{rs} \leq \text{XM} \tag{2.16}$$

and only those TE^e_{rs} and TE^o_{rs} modes for all $\{rs\}$ such that

$$x'_{rs} \leq \text{XM}. \tag{2.17}$$

However, if we did so, the maximum value of s in (2.17) would not necessarily be the same as the maximum value of s in (2.16). The only difference between the $\{x_{rs}, x'_{rs}\}$ allowed by (2.16) and (2.17) and the $\{x_{rs}, x'_{rs}\}$ in (2.14) is that we could have

$$x'_{0s_{\max}} > \text{XM} \tag{2.18}$$

or

$$x_{rs_{\max}} > \text{XM} \tag{2.19}$$

in (2.14). The root (2.18) is not allowed by (2.17), and the root (2.19) is not allowed by (2.16).

Whereas the previously described first seven variables in the first read statement are real, the last two variables in this read statement are complex. These complex variables are ZL1 and ZL2 given by

$$\text{ZL1} = Z_1 Y^{\text{TE}}_{10} \tag{2.20}$$

$$\text{ZL2} = Z_2 Y^{\text{TE}}_{10} \tag{2.21}$$

where Y_{10}^{TE} is the admittance of the TE_{10} rectangular waveguide mode (see eq. (A.25) of [1]):

$$Y_{10}^{\mathrm{TE}} = \frac{\gamma_{10}}{j\omega\mu} \tag{2.22}$$

where ω is the angular frequency and (see eq. (A.12) of [1])

$$\gamma_{10} = \sqrt{k_{10}^2 - k^2} \tag{2.23}$$

where, according to (2.6),

$$k_{10} = \frac{\pi}{b}. \tag{2.24}$$

In (2.23), k is the wavenumber given by

$$k = \omega\sqrt{\mu\epsilon}\ . \tag{2.25}$$

In view of (2.24), substitution of (2.23) into (2.22) gives

$$Y_{10}^{\mathrm{TE}} = -\frac{j}{\eta}\sqrt{\left(\frac{\pi}{kb}\right)^2 - 1} \tag{2.26}$$

where η is the intrinsic impedance given by

$$\eta = \sqrt{\frac{\mu}{\epsilon}}\ . \tag{2.27}$$

Substituting (2.26) into (2.20) and (2.21), we obtain

$$\mathrm{ZL1} = -\frac{jZ_1}{\eta}\sqrt{\left(\frac{\pi}{kb}\right)^2 - 1} \tag{2.28}$$

$$\mathrm{ZL2} = -\frac{jZ_2}{\eta}\sqrt{\left(\frac{\pi}{kb}\right)^2 - 1}\ . \tag{2.29}$$

The first seven variables in the first read statement are obviously dimensionless. The variables ZL1 of (2.28) and ZL2 of (2.29) are dimensionless because Z_1/η and Z_2/η are dimensionless. Therefore, all the variables in the first read statement are dimensionless.

The variables KAM, BKA0, and DBKA in the second read statement are such that the waveguide mode converter problem is solved KAM times, once for each of the following values of ka:

$$ka = \mathrm{BKA0} + (\mathrm{KA} - 1) * \mathrm{DBKA},\ \mathrm{KA} = 1, 2, 3, \cdots, \mathrm{KAM}. \tag{2.30}$$

The variables KE3M, NPHI, and NZ in the second read statement and {KE3(I), I = $1, 2, \cdots$, KE3M} in the third read statement control the calculation and the writing out of the magnitudes of the ϕ- and z-components of the normalized electric field in the apertures A_1 and A_2. These magnitudes are

$$\frac{|E_\phi^{(A1)}|}{|\underline{E}_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}}, \quad \frac{|E_z^{(A1)}|}{|\underline{E}_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}}, \quad \frac{|E_\phi^{(A2)}|}{|\underline{E}_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}}, \text{ and } \frac{|E_z^{(A2)}|}{|\underline{E}_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}}. \tag{2.31}$$

Here, $E_\phi^{(A1)}$ is the ϕ-component of the electric field at

$$(\phi, z) = \left(\left\{\pi + \phi_o\left(-1 + 2\frac{\mathrm{J}-1}{\mathrm{NPHI}-1}\right), \mathrm{J} = 1, 2, \cdots, \mathrm{NPHI}\right\}, 0\right) \tag{2.32}$$

in aperture A_1, and $E_z^{(A1)}$ is the z-component of the electric field at

$$(\phi, z) = \left(\pi, \left\{\frac{c}{2}\left(-1 + 2\frac{\mathrm{J}-1}{\mathrm{NZ}-1}\right), \mathrm{J} = 1, 2, \cdots, \mathrm{NZ}\right\}\right) \tag{2.33}$$

in aperture A_1. Moreover, $E_\phi^{(A2)}$ is the ϕ-component of the electric field at

$$(\phi, z) = \left(\left\{\phi_o\left(-1 + 2\frac{\mathrm{J}-1}{\mathrm{NPHI}-1}\right), \mathrm{J} = 1, 2, \cdots, \mathrm{NPHI}\right\}, 0\right) \tag{2.34}$$

in aperture A_2, and $E_z^{(A2)}$ is the z-component of the electric field at

$$(\phi, z) = \left(0, \left\{\frac{c}{2}\left(-1 + 2\frac{\mathrm{J}-1}{\mathrm{NZ}-1}\right), \mathrm{J} = 1, 2, \cdots, \mathrm{NZ}\right\}\right) \tag{2.35}$$

in aperture $A2$. Finally, $|\underline{E}_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}$ is the root mean square value of the transverse part of $\underline{E}_{01}^{\mathrm{TMe+}}$ taken over the cross section of the circular waveguide at $z = 0$. Here, $\underline{E}_{01}^{\mathrm{TMe+}}$ is the electric field of the TM_{01}^{e+} wave† of unit amplitude traveling in the z-direction in the circular waveguide. Otherwise stated, $\underline{E}_{01}^{\mathrm{TMe+}}$ is the electric field that would exist in the circular waveguide if there were no reflections, i.e., if the apertures A_1 and A_2 were closed with perfect conductors and if the circular waveguide were to extend to $z = \infty$ instead of being terminated by a perfectly conducting wall at $z = L_3$.

The calculation and the writing out of the aperture field magnitudes (2.31) are controlled by the input array KE3 according to

†The superscript "e" in TM_{01}^{e+} indicates that the z-component of the electric field of the wave is even in ϕ. The superscript "+" in TM_{01}^{e+} indicates that the wave travels in the $+z$-direction.

```
      KAE=1
      DO 48 KA=1,KAM
      BKA=BKA0+(KA-1)*DBKA
C     FORTRAN STATEMENTS TO SOLVE THE WAVEGUIDE MODE CONVERTER
C     PROBLEM FOR THE ABOVE VALUE OF BKA.
      IF(KA.NE.KE3(KAE)) GO TO 48
      KAE=KAE+1
C     FORTRAN STATEMENTS TO CALCULATE AND TO WRITE OUT THE
C     APERTURE FIELD MAGNITUDES (2.31).
   48 CONTINUE
```

where

$$\mathrm{BKA} = ka. \tag{2.36}$$

According to the above FORTRAN statements, if one choses

$$1 \leq \mathrm{KE3}(1) \leq \mathrm{KE3}(2) \leq \mathrm{KE3}(3) \cdots, \mathrm{KE3}(\mathrm{KE3M}) \tag{2.37}$$

where

$$\mathrm{KE3M} \leq \mathrm{KAM} \tag{2.38}$$

and

$$\mathrm{KE3}(\mathrm{KE3M}) \geq \mathrm{KAM}, \tag{2.39}$$

then the aperture field magnitudes (2.31) are calculated and written out for

$$ka = \{\mathrm{BKA0} + (\mathrm{KE3(J)} - 1) * \mathrm{DBKA},\ \mathrm{J} = 1, 2, \cdots, \mathrm{KE3MM}\} \tag{2.40}$$

where

$$\mathrm{KE3MM} = \begin{cases} \mathrm{KE3M} - 1, & \mathrm{KE3}(\mathrm{KE3M}) > \mathrm{KAM} \\ \mathrm{KE3M}, & \mathrm{KE3}(\mathrm{KE3M}) = \mathrm{KAM}. \end{cases} \tag{2.41}$$

2.1.2 The Second Module of Input Data

The second module of input data is read by means of statements in the subroutine BESIN. One who merely uses the computer program does not have to concern oneself with this module of input data because it is always the same in the sense that one will never have to change the numerical values contained in it.

2.1.3 Sample Input Data

When the computer program was run with the input data listed in Section 2.1.3, the output data were those listed in Section 2.2.2. These input data are for the structure of Fig. 2 with

$$\frac{b}{a} = 1.1 \tag{2.42}$$

$$\frac{c}{a} = 0.5 \tag{2.43}$$

$$L_3 = 0.5\left[\lambda_{01}^{\mathrm{TM}}\right]_{ka=2.95} \tag{2.44}$$

$$Z_1 = \frac{1}{Y_{10}^{\mathrm{TE}}} \tag{2.45}$$

$$Z_2 = \frac{1}{Y_{10}^{\mathrm{TE}}} \tag{2.46}$$

$$ka = 2.95. \tag{2.47}$$

In (2.44), $\left[\lambda_{01}^{\mathrm{TM}}\right]_{ka=2.95}$ is the wavelength of the TM circular waveguide mode when $ka = 2.95$. According to eq. (8.4) of [2],

$$\left[\lambda_{01}^{\mathrm{TM}}\right]_{ka=2.95} = 3.67738806a \tag{2.48}$$

so that

$$L_3 = 1.838694a. \tag{2.49}$$

In (2.45) and (2.46), Y_{10}^{TE} is the characteristic admittance of the TE_{10} circular waveguide mode so that Z_1 and Z_2 are matched loads. Consequently, the electromagnetic field in the circular waveguide will not depend on either L_1 or L_2 provided that L_1 and L_2 are large enough so that any evanescent wave emanating from either the termination at $x = -L_1$ or that at $x = L_2$ will have negligibly small amplitude upon arrival at the pertinent aperture in the circular waveguide. The single value ka of (2.47) was obtained by setting

$$\mathrm{KAM} = 1 \tag{2.50}$$

$$\mathrm{BKA0} = 2.95. \tag{2.51}$$

Because KAM = 1, the value of DBKA is inconsequential. The values of the variables BKM, XM, NPHI, NZ, KE3M, and KE3(1) in the sample input are given by

$$\mathrm{BKM} = 15. \tag{2.52}$$
$$\mathrm{XM} = 40. \tag{2.53}$$
$$\mathrm{NPHI} = 81 \tag{2.54}$$
$$\mathrm{NZ} = 21 \tag{2.55}$$
$$\mathrm{KE3M} = 1 \tag{2.56}$$
$$\mathrm{KE3(1)} = 1. \tag{2.57}$$

Because of (2.56) and (2.57), the magnitudes of the ϕ- and z-components of the normalized electric field in the apertures A_1 and A_2 will appear in the output data.

Listing of the first module of the sample input data

```
 0.1100000D+01 0.5000000D+00 0.4000000D+02 0.4000000D+02
 0.1838694D+01 0.1500000D+02 0.4000000D+02 0.1000000D+01
 0.0000000D+00 0.1000000D+01 0.0000000D+00
   1 0.2950000D+01 0.0000000D+00    1   81   21
   1
```

Listing of the second module of the sample input data

```
ROOTS OF BESSEL FUNCTIONS ((X(N,S),S=1,50), N=1,21)
   2.40482556    5.52007811    8.65372791   11.79153444   14.93091771
  18.07106397   21.21163663   24.35247153   27.49347913   30.63460647
  33.77582021   36.91709835   40.05842576   43.19979171   46.34118837
  49.48260990   52.62405184   55.76551076   58.90698393   62.04846919
  65.18996480   68.33146933   71.47298160   74.61450064   77.75602563
  80.89755587   84.03909078   87.18062984   90.32217264   93.46371878
  96.60526795   99.74681986  102.88837425  106.02993092  109.17148965
 112.31305028  115.45461265  118.59617663  121.73774209  124.87930891
 128.02087701  131.16244628  134.30401664  137.44558802  140.58716035
 143.72873357  146.87030763  150.01188246  153.15345802  156.29503427
   3.83170597    7.01558667   10.17346814   13.32369194   16.47063005
  19.61585851   22.76008438   25.90367209   29.04682853   32.18967991
  35.33230755   38.47476623   41.61709421   44.75931900   47.90146089
  51.04353518   54.18555364   57.32752544   60.46945785   63.61135670
```

66.75322673	69.89507184	73.03689523	76.17869958	79.32048718
82.46225991	85.60401944	88.74576714	91.88750425	95.02923181
98.17095073	101.31266182	104.45436579	107.59606326	110.73775478
113.87944085	117.02112190	120.16279833	123.30447049	126.44613870
129.58780325	132.72946439	135.87112236	139.01277739	142.15442966
145.29607935	148.43772662	151.57937163	154.72101452	157.86265540
5.13562230	8.41724414	11.61984117	14.79595178	17.95981949
21.11699705	24.27011231	27.42057355	30.56920450	33.71651951
36.86285651	40.00844673	43.15345378	46.29799668	49.44216411
52.58602351	55.72962705	58.87301577	62.01622236	65.15927319
68.30218978	71.44498987	74.58768817	77.73029706	80.87282695
84.01528671	87.15768394	90.30002515	93.44231602	96.58456145
99.72676573	102.86893265	106.01106552	109.15316729	112.29524056
115.43728766	118.57931068	121.72131148	124.86329174	128.00525297
131.14719653	134.28912367	137.43103552	140.57293310	143.71481735
146.85668912	149.99854919	153.14039829	156.28223708	159.42406617
6.38016190	9.76102313	13.01520072	16.22346616	19.40941523
22.58272959	25.74816670	28.90835078	32.06485241	35.21867074
38.37047243	41.52071967	44.66974312	47.81778569	50.96502991
54.11161557	57.25765160	60.40322414	63.54840218	66.69324167
69.83778844	72.98208040	76.12614918	79.27002139	82.41371955
85.55726287	88.70066784	91.84394868	94.98711773	98.13018573
101.27316212	104.41605517	107.55887218	110.70161965	113.84430334
116.98692838	120.12949939	123.27202050	126.41449544	129.55692756
132.69931991	135.84167526	138.98399610	142.12628474	145.26854326
148.41077358	151.55297745	154.69515649	157.83731217	160.97944587
7.58834243	11.06470949	14.37253667	17.61596605	20.82693296
24.01901952	27.19908777	30.37100767	33.53713771	36.69900113
39.85762730	43.01373772	46.16785351	49.32036069	52.47155140
55.62165091	58.77083574	61.91924620	65.06699526	68.21417486
71.36086067	74.50711546	77.65299182	80.79853407	83.94377989
87.08876147	90.23350652	93.37803898	96.52237969	99.66654682
102.81055633	105.95442227	109.09815708	112.24177180	115.38527625
118.52867922	121.67198858	124.81521142	127.95835412	131.10142245
134.24442164	137.38735644	140.53023118	143.67304979	146.81581590
149.95853279	153.10120351	156.24383085	159.38641736	162.52896543
8.77148382	12.33860420	15.70017408	18.98013388	22.21779990
25.43034115	28.62661831	31.81171672	34.98878129	38.15986856
41.32638325	44.48931912	47.64939981	50.80716520	53.96302656
57.11730278	60.27024507	63.42205405	66.57289189	69.72289116
72.87216130	76.02079343	79.16886409	82.31643800	85.46357030
88.61030824	91.75669254	94.90275852	98.04853691	101.19405463
104.33933531	107.48439983	110.62926667	113.77395226	116.91847126
120.06283680	123.20706064	126.35115339	129.49512461	132.63898297

135.78273630	138.92639176	142.06995586	145.21343453	148.35683321
151.50015689	154.64341015	157.78659721	160.92972194	164.07278793
9.93610952	13.58929017	17.00381967	20.32078921	23.58608444
26.82015198	30.03372239	33.23304176	36.42201967	39.60323942
42.77848161	45.94901600	49.11577372	52.27945390	55.44059207
58.59960563	61.75682490	64.91251478	68.06689027	71.22012770
74.37237311	77.52374850	80.67435660	83.82428452	86.97360663
90.12238683	93.27068030	96.41853497	99.56599267	102.71309005
105.85985938	109.00632919	112.15252478	115.29846869	118.44418103
121.58967984	124.73498132	127.88010007	131.02504929	134.16984093
137.31448584	140.45899391	143.60337414	146.74763477	149.89178335
153.03582678	156.17977144	159.32362318	162.46738741	165.61106911
11.08637002	14.82126873	18.28758283	21.64154102	24.93492789
28.19118846	31.42279419	34.63708935	37.83871738	41.03077369
44.21540851	47.39416576	50.56818468	53.73832537	56.90524999
60.06947700	63.23141837	66.39140576	69.54970927	72.70655117
75.86211608	79.01655863	82.17000939	85.32257933	88.47436342
91.62544340	94.77589002	97.92576484	101.07512166	104.22400772
107.37246468	110.52052942	113.66823467	116.81560966	119.96268048
123.10947057	126.25600101	129.40229081	132.54835716	135.69421567
138.83988051	141.98536460	145.13067973	148.27583667	151.42084532
154.56571475	157.71045331	160.85506870	163.99956802	167.14395783
12.22509226	16.03777419	19.55453643	22.94517313	26.26681464
29.54565967	32.79580004	36.02561506	39.24044800	42.44388774
45.63844418	48.82593038	52.00769146	55.18474794	58.35788903
61.52773517	64.69478124	67.85942699	71.02199904	74.18276693
77.34195516	80.49975227	83.65631779	86.81178765	89.96627840
93.11989054	96.27271125	99.42481648	102.57627274	105.72713851
108.87746543	112.02729929	115.17668080	118.32564630	121.47422834
124.62245614	127.77035603	130.91795177	134.06526490	137.21231495
140.35911970	143.50569535	146.65205670	149.79821731	152.94418961
156.08998503	159.23561409	162.38108649	165.52641118	168.67159646
13.35430048	17.24122038	20.80704779	24.23388526	27.58374896
30.88537897	34.15437792	37.40009998	40.62855372	43.84380142
47.04870074	50.24532696	53.43522716	56.61958027	59.79930163
62.97511353	66.14759402	69.31721152	72.48434982	75.64932654
78.81240687	81.97381406	85.13373741	88.29233855	91.44975632
94.60611070	97.76150589	100.91603286	104.06977136	107.22279165
110.37515584	113.52691905	116.67813038	119.82883371	122.97906837
126.12886972	129.27826963	132.42729694	135.57597775	138.72433580
141.87239270	145.02016815	148.16768018	151.31494531	154.46197871
157.60879431	160.75540495	163.90182250	167.04805790	170.19412130
14.47550069	18.43346367	22.04698536	25.50945055	28.88737506
32.21185620	35.49990921	38.76180702	42.00419024	45.23157410

48.44715139	51.65325167	54.85161908	58.04358793	61.23019798
64.41227241	67.59047207	70.76533400	73.93729938	77.10673425
80.27394491	83.43918980	86.60268848	89.76462879	92.92517238
96.08445917	99.24261087	102.39973390	105.55592171	108.71125670
111.86581184	115.01965195	118.17283486	121.32541227	124.47743061
127.62893164	130.77995306	133.93052898	137.08069034	140.23046527
143.37987941	146.52895616	149.67771696	152.82618144	155.97436765
159.12229220	162.26997039	165.41741635	168.56464316	171.71166291
15.58984788	19.61596690	23.27585373	26.77332255	30.17906118
33.52636408	36.83357134	40.11182327	43.36836095	46.60813268
49.83465351	53.05049896	56.25760472	59.45745691	62.65121739
65.83980880	69.02397393	72.20431796	75.38133933	78.55545246
81.72700494	84.89629058	88.06355952	91.22902609	94.39287509
97.55526669	100.71634047	103.87621862	107.03500857	110.19280522
113.34969265	116.50574569	119.66103113	122.81560878	125.96953239
129.12285034	132.27560638	135.42784009	138.57958743	141.73088111
144.88175097	148.03222428	151.18232600	154.33207905	157.48150449
160.63062171	163.77944859	166.92800168	170.07629624	173.22434647
16.69824993	20.78990636	24.49488504	28.02670995	31.45996004
34.82998699	38.15637750	41.45109231	44.72194354	47.97429353
51.21196700	54.43777693	57.65384481	60.86180468	64.06293782
67.25826456	70.44860840	73.63464196	76.81692044	79.99590644
83.17198872	86.34549652	89.51671063	92.68587205	95.85318889
99.01884180	102.18298842	105.34576695	108.50729906	111.66769230
114.82704216	117.98543364	121.14294273	124.29963753	127.45557930
130.61082324	133.76541929	136.91941271	140.07284461	143.22575244
146.37817038	149.53012967	152.68165898	155.83278459	158.98353070
162.13391960	165.28397186	168.43370649	171.58314112	174.73229205
17.80143515	21.95624407	25.70510305	29.27063044	32.73105331
36.12365767	39.46920683	42.78043927	46.06571091	49.33078010
52.57976906	55.81571988	59.04093404	62.25718939	65.46588380
68.66813322	71.86484051	75.05674474	78.24445722	81.42848829
84.60926767	87.78715986	90.96247628	94.13548463	97.30641638
100.47547280	103.64282970	106.80864143	109.97304405	113.13615796
116.29809015	119.45893600	122.61878084	125.77770123	128.93576606
132.09303754	135.24957191	138.40542024	141.56062893	144.71524029
147.86929295	151.02282228	154.17586070	157.32843800	160.48058160
163.63231679	166.78366690	169.93465353	173.08529669	176.23561493
18.89999795	23.11577835	26.90736898	30.50595016	33.99318498
37.40818513	40.77282785	44.10059057	47.40034778	50.67823695
53.93866621	57.18489860	60.41940985	63.64411751	66.86053301
70.06986583	73.27309662	76.47102976	79.66433188	82.85356054
86.03918598	89.22160778	92.40116781	95.57816038	98.75284036
101.92542960	105.09612218	108.26508867	111.43247958	114.59842828

117.76305340	120.92646079	124.08874523	127.24999185	130.41027733
133.56967093	136.72823538	139.88602761	143.04309944	146.19949814
149.35526690	152.51044527	155.66506956	158.81917313	161.97278672
165.12593867	168.27865519	171.43096051	174.58287710	177.73442583
19.99443063	24.26918003	28.10241523	31.73341334	35.24708679
38.68427639	42.06791700	45.41218961	48.72646412	52.01724128
55.28920415	58.54582890	61.78975990	65.02305025	68.24732200
71.46387589	74.67376871	77.87786897	81.07689772	84.27145907
87.46206333	90.64914480	93.83307557	97.01417644	100.19272555
103.36896532	106.54310808	109.71534063	112.88582801	116.05471661
119.22213673	122.38820479	125.55302510	128.71669145	131.87928838
135.04089230	138.20157249	141.36139187	144.52040775	147.67867242
150.83623371	153.99313547	157.14941792	160.30511811	163.46027013
166.61490548	169.76905326	172.92274044	176.07599201	179.22883118
21.08514611	25.41701901	29.29087070	32.95366489	36.49339791
39.95255349	43.35507320	46.71580944	50.04460602	53.34831233
56.63187594	59.89897873	63.15242819	66.39440904	69.62665088
72.85054351	76.06721817	79.27760620	82.48248211	85.68249584
88.87819724	92.07005490	95.25847087	98.44379219	101.62632016
104.80631766	107.98401523	111.15961587	114.33329914	117.50522445
120.67553386	123.84435435	127.01179984	130.17797281	133.34296574
136.50686227	139.66973827	142.83166272	145.99269847	149.15290292
152.31232861	155.47102371	158.62903248	161.78639564	164.94315076
168.09933252	171.25497299	174.41010189	177.56474681	180.71893336
22.17249462	26.55978414	30.47327995	34.16726785	37.73268052
41.21356706	44.63482975	48.01196294	51.35526465	54.67191918
57.96712883	61.24477410	64.50782040	67.75858011	70.99888749
74.23021912	77.45377900	80.67055998	83.88138905	87.08696117
90.28786514	93.48460342	96.67760765	99.86725087	103.05385733
106.23771026	109.41905827	112.59812055	115.77509113	118.95014243
122.12342824	125.29508615	128.46523971	131.63400014	134.80146786
137.96773380	141.13288046	144.29698294	147.46010968	150.62232324
153.78368091	156.94423525	160.10403458	163.26312338	166.42154271
169.57933048	172.73652180	175.89314920	179.04924287	182.20483087
23.25677609	27.69789835	31.65011815	35.37471722	38.96543205
42.46780721	45.90766387	49.30111134	52.65888365	55.98848722
59.29536994	62.58360418	65.85630828	69.11591850	72.36437087
75.60322657	78.83376063	82.05702611	85.27390141	88.48512576
91.69132626	94.89303872	98.09072402	101.28478091	104.47555635
107.66335375	110.84843970	114.03104944	117.21139142	120.38965105
123.56599381	126.74056792	129.91350652	133.08492960	136.25494557
139.42365263	142.59114000	145.75748887	148.92277333	152.08706115
155.25041440	158.41289006	161.57454055	164.73541416	167.89555548
171.05500571	174.21380299	177.37198271	180.52957773	183.68661859

24.33824962	28.83173035	32.82180276	36.57645076	40.19209510
43.71571242	47.17400457	50.58367114	53.95586528	57.29840365
60.61697113	63.91582558	67.19823350	70.46675142	73.72341433
76.96986585	80.20745037	83.43727983	86.66028299	89.87724253
93.08882319	96.29559365	99.49804359	102.69659716	105.89162379
109.08344685	112.27235076	115.45858672	118.64237751	121.82392144
125.00339567	128.18095899	131.35675416	134.53090991	137.70354262
140.87475783	144.04465143	147.21331077	150.38081561	153.54723893
156.71264761	159.87710310	163.04066196	166.20337631	169.36529427
172.52646038	175.68691587	178.84669901	182.00584532	185.16438789
25.41714081	29.96160379	33.98870279	37.77285784	41.41306551
44.95767675	48.43423920	51.86001993	55.24657561	58.60202207
61.93227307	65.24176599	68.53391094	71.81138120	75.07630808
78.33041549	81.57511555	84.81157774	88.04078020	91.26354816
94.48058339	97.69248687	100.89977667	104.10290200	107.30225447
110.49817724	113.69097235	116.88090682	120.06821766	123.25311603
126.43579068	129.61641094	132.79512915	135.97208276	139.14739615
142.32118214	145.49354333	148.66457323	151.83435729	155.00297369
158.17049419	161.33698470	164.50250593	167.66711385	170.83086019
173.99379280	177.15595604	180.31739107	183.47813615	186.63822689

ROOTS OF DERIVATIVES OF BESSEL FUNCTIONS ((XP(N,S),S=1,50),N=1,21)

3.83170597	7.01558667	10.17346814	13.32369194	16.47063005
19.61585851	22.76008438	25.90367209	29.04682853	32.18967991
35.33230755	38.47476623	41.61709421	44.75931900	47.90146089
51.04353518	54.18555364	57.32752544	60.46945785	63.61135670
66.75322673	69.89507184	73.03689523	76.17869958	79.32048718
82.46225991	85.60401944	88.74576714	91.88750425	95.02923181
98.17095073	101.31266182	104.45436579	107.59606326	110.73775478
113.87944085	117.02112190	120.16279833	123.30447049	126.44613870
129.58780325	132.72946439	135.87112236	139.01277739	142.15442966
145.29607935	148.43772662	151.57937163	154.72101452	157.86265540
1.84118378	5.33144277	8.53631637	11.70600490	14.86358863
18.01552786	21.16436986	24.31132686	27.45705057	30.60192297
33.74618290	36.88998741	40.03344405	43.17662897	46.31959756
49.46239114	52.60504111	55.74757179	58.89000230	62.03234787
65.17462080	68.31683113	71.45898711	74.60109561	77.74316241
80.88519235	84.02718959	87.16915764	90.31109957	93.45301801
96.59491525	99.73679330	102.87865391	106.02049864	109.16232885
112.30414577	115.44595048	118.58774396	121.72952706	124.87130058
128.01306522	131.15482162	134.29657036	137.43831196	140.58004691
143.72177563	146.86349853	150.00521597	153.14692830	156.28863581
3.05423693	6.70613319	9.96946782	13.17037086	16.34752232
19.51291278	22.67158177	25.82603714	28.97767277	32.12732702
35.27553505	38.42265482	41.56893494	44.71455353	47.85964161

51.00429767	54.14859724	57.29259919	60.43635008	63.57988724
66.72324095	69.86643601	73.00949296	76.15242892	79.29525830
82.43799331	85.58064435	88.72322036	91.86572905	95.00817710
98.15057035	101.29291390	104.43521224	107.57746933	110.71968869
113.86187345	117.00402639	120.14615001	123.28824656	126.43031806
129.57236632	132.71439301	135.85639961	138.99838749	142.14035790
145.28231196	148.42425071	151.56617512	154.70808604	157.84998430
4.20118894	8.01523660	11.34592431	14.58584829	17.78874787
20.97247694	24.14489743	27.31005793	30.47026881	33.62694918
36.78102068	39.93310862	43.08365266	46.23297108	49.38130009
52.52881874	55.67566523	58.82194800	61.96775330	65.11315060
68.25819654	71.40293768	74.54741272	77.69165407	80.83568905
83.97954092	87.12322953	90.26677197	93.41018304	96.55347558
99.69666083	102.83974863	105.98274768	109.12566565	112.26850936
115.41128489	118.55399765	121.69665253	124.83925389	127.98180569
131.12431149	134.26677452	137.40919772	140.55158376	143.69393509
146.83625393	149.97854233	153.12080216	156.26303515	159.40524288
5.31755313	9.28239629	12.68190844	15.96410704	19.19602880
22.40103227	25.58975968	28.76783622	31.93853934	35.10391668
38.26531699	41.42366650	44.57962314	47.73366752	50.88615915
54.03737242	57.18752046	60.33677140	63.48525967	66.63309405
69.78036353	72.92714162	76.07348960	79.21945893	82.36509317
85.51042944	88.65549957	91.80033100	94.94494751	98.08936983
101.23361611	104.37770230	107.52164246	110.66544908	113.80913324
116.95270484	120.09617273	123.23954486	126.38282839	129.52602978
132.66915487	135.81220898	138.95519692	142.09812309	145.24099151
148.38380585	151.52656948	154.66928549	157.81195673	160.95458583
6.41561638	10.51986087	13.98718863	17.31284249	20.57551452
23.80358148	27.01030790	30.20284908	33.38544390	36.56077769
39.73064023	42.89627316	46.05856627	49.21817461	52.37559153
55.53119588	58.68528359	61.83808923	64.98980119	68.14057257
71.29052908	74.43977491	77.58839718	80.73646930	83.88405355
87.03120316	90.17796384	93.32437513	96.47047134	99.61628246
102.76183477	105.90715140	109.05225282	112.19715718	115.34188065
118.48643767	121.63084118	124.77510283	127.91923309	131.06324144
134.20713647	137.35092598	140.49461707	143.63821620	146.78172930
149.92516179	153.06851864	156.21180443	159.35502337	162.49817934
7.50126614	11.73493595	15.26818146	18.63744301	21.93171502
25.18392560	28.40977636	31.61787572	34.81339298	37.99964090
41.17884947	44.35257920	47.52195691	50.68781778	53.85079464
57.01137608	60.16994561	63.32680859	66.48221126	69.63635446
72.78940366	75.94149646	79.09274823	82.24325646	85.39310406
88.54236204	91.69109156	94.83934557	97.98717018	101.13460569
104.28168751	107.42844688	110.57491145	113.72110579	116.86705181

120.01276912	123.15827530	126.30358618	129.44871605	132.59367782
135.73848322	138.88314289	142.02766654	145.17206303	148.31634048
151.46050631	154.60456734	157.74852986	160.89239965	164.03618206
8.57783649	12.93238624	16.52936588	19.94185337	23.26805293
26.54503206	29.79074858	33.01517864	36.22438055	39.42227458
42.61152217	45.79399966	48.97107095	52.14375297	55.31282033
58.47887403	61.64238785	64.80374053	67.96323864	71.12113304
74.27763106	77.43290562	80.58710208	83.74034356	86.89273506
90.04436665	93.19531609	96.34565085	99.49542974	102.64470429
105.79351984	108.94191644	112.08992959	115.23759089	118.38492855
121.53196784	124.67873146	127.82523986	130.97151151	134.11756314
137.26340993	140.40906571	143.55454307	146.69985354	149.84500765
152.99001507	156.13488470	159.27962473	162.42424269	165.56874557
9.64742165	14.11551891	17.77401237	21.22906262	24.58719749
27.88926943	31.15532656	34.39662855	37.62007804	40.83017868
44.03001034	47.22175847	50.40702097	53.58699544	56.76259848
59.93454431	63.10339820	66.26961367	69.43355902	72.59553655
75.75579686	78.91454945	82.07197091	85.22821114	88.38339823
91.53764234	94 [illegible]3869	97.84367007	100.99560878	104.14691826
107.29765441	1[illegible]0 44786667	113.59759895	116.74689035	119.89577586
123.04428682	12[illegible].19245144	129.34029514	132.48784093	135.63510964
138.782120[illegible]2	141.92888993	145.07543449	148.22176832	151.36790460
154.513[illegible]46	157.65963202	160.80524457	163.95070256	167.09601476
10.71143397	15.28673767	19.00459354	22.50139873	25.89127728
29 21856350	32.50524735	35.76379293	39.00190281	42.22463843
45.43548310	48.63692265	51.83078393	55.01844255	58.20095582
61.37915081	64.55368443	67.72508544	70.89378457	74.06013637
77.22443549	80.38692888	83.54782516	86.70730178	89.86551073
93.02258289	96.17863168	99.33375580	102.48804162	105.64156505
108.79439302	111.94658487	115.09819332	118.24926541	121.39984324
124.54996459	127.69966351	130.84897070	133.99791395	137.14651851
140.29480729	143.44280122	146.59051940	149.73797930	152.88519695
156.03218709	159.17896326	162.32553797	165.47192277	168.61812831
11.77087667	16.44785275	20.22303141	23.76071586	27.18202153
30.53450475	33.84196578	37.11800042	40.37106891	43.60676490
46.82895945	50.04042897	53.[illegible]4322321	56.43889206	59.62863131
62.81337965	65.99388505	69.17075142	72.34447202	75.51545393
78.68403628	81.85050394	85.01509806	88.17802420	91.33945869
94.49955372	97.65844131	100.81623660	103.97304045	107.12894162
110.28401853	113.43834073	116.59197013	119.74496202	122.89736594
126.04922639	129.20058350	132.35147351	135.50192928	138.65198060
141.80165465	144.95097616	148.09996779	151.24865026	154.39704260
157.54516231	160.69302549	163.84064702	166.98804062	170.13521901
12.82649123	17.60026656	21.43085424	25.00851870	28.46085728

31.83842446	35.16671443	38.46038872	41.72862556	44.97752625
48.21133384	51.43310517	54.64510624	57.84905686	61.04628851
64.23784974	67.42457850	70.60715320	73.78612937	76.96196660
80.13504857	83.30569829	86.47418969	89.64075672	92.80560044
95.96889462	99.13079026	102.29141924	105.45089726	108.60932625
111.76679640	114.92338778	118.07917175	121.23421211	124.38856609
127.54228514	130.69541570	133.84799977	137.00007541	140.15167724
143.30283677	146.45358278	149.60394161	152.75393739	155.90359227
159.05292666	162.20195932	165.35070763	168.49918760	171.64741409
13.87884307	18.74509092	22.62930030	26.24604777	29.72897817
33.13144995	36.48054830	39.79194072	43.07548680	46.33777210
49.58339642	52.81568683	56.03711869	59.24957708	62.45452600
65.65312168	68.84629065	72.03478491	75.21922169	78.40011274
81.57788618	84.75290311	87.92547036	91.09585040	94.26426915
97.43092221	100.59597977	103.75959067	106.92188567	110.08298013
113.24297620	116.40196472	119.56002670	122.71723465	125.87365369
129.02934241	132.18435374	135.33873558	138.49253142	141.64578078
144.79851973	147.95078119	151.10259532	154.25398977	157.40498996
160.55561928	163.70589931	166.85584997	170.00548971	173.15483560
14.92837449	19.88322436	23.81938909	27.47433975	30.98739433
34.41454566	37.78437851	41.11351238	44.41245452	47.68825285
50.94584925	54.18883107	57.41987615	60.64103003	63.85388583
67.05970512	70.25950133	73.45409890	76.64417598	79.83029598
83.01293114	86.19248048	89.36928364	92.54363161	95.71577527
98.88593209	102.05429160	105.22101975	108.38626251	111.55014879
114.71279291	117.87429661	121.03475072	124.19423664	127.35282750
130.51058919	133.66758129	136.82385774	139.97946755	143.13445532
146.28886175	149.44272406	152.59607631	155.74894981	158.90137331
162.05337332	165.20497428	168.35619880	171.50706779	174.65760064
15.97543881	21.01540493	25.00197150	28.69427122	32.23696941
35.68854409	39.07899819	42.42585443	45.74023678	49.02963506
52.29931939	55.55312778	58.79393376	62.02393848	65.24486077
68.45806499	71.66464970	74.86551046	78.06138542	81.25288898
84.44053706	87.62476630	90.80594897	93.98440448	97.16040861
100.33420074	103.50598977	106.67595882	109.84426916	113.01106338
116.17646804	119.34059592	122.50354778	125.66541401	128.82627588
131.98620672	135.14527282	138.30353431	141.46104582	144.61785714
147.77401370	150.92955708	154.08452540	157.23895364	160.39287403
163.54631625	166.69930769	169.85187372	173.00403778	176.15582166
17.02032327	22.14224735	26.17776620	29.90659108	33.47844849
36.95416965	40.36510275	43.72962958	47.05946240	50.36251400
53.64436[illegible]64	56.90910879	60.15979410	63.39877760	66.62790029
69.84862681	73.06213892	76.26940192	79.47121293	82.66823660
85.86103195	89.05007287	92.23576396	95.41845289	98.59844022

101.77598720	104.95132204	108.12464507	111.29613289	114.46594181
117.63421073	120.80106350	123.96661091	127.13095242	130.29417754
133.45636709	136.61759421	139.77792526	142.93742061	146.09613529
149.25411956	152.41141945	155.56807718	158.72413153	161.87961824
165.03457024	168.18901794	171.34298952	174.49651104	177.64960672
18.06326499	23.26426978	27.34738651	31.11194494	34.71247959
38.21205723	41.64330585	45.02542621	48.37069283	51.68742381
54.98150733	58.25725555	61.51791395	64.76598075	68.00341570
71.23178115	74.45234013	77.66612622	80.87399440	84.07665874
87.27472076	90.46869112	93.65900642	96.84604234	100.03012405
103.21153453	106.39052128	109.56730178	112.74206794	115.91498982
119.08621866	122.25588944	125.42412303	128.59102801	131.75670219
134.92123395	138.08470332	141.24718298	144.40873909	147.56943199
150.72931686	153.88844422	157.04686045	160.20460821	163.36172678
166.51825242	169.67421863	172.82965642	175.98459455	179.13905971
19.10446224	24.38191370	28.51136068	32.31089394	35.93963034
39.46276685	42.91415216	46.31376949	49.67443171	53.00484590
56.31119165	59.59800556	62.86870984	66.12594446	69.37178453
72.60788756	75.83559590	79.05600992	82.27004135	85.47845278
88.68188756	91.88089261	95.07593614	98.26742155	101.45569839
104.64107123	107.82380672	111.00413944	114.18227656	117.35840186
120.53267893	123.70525388	126.87625768	130.04580806	133.21401118
136.38096300	139.54675051	142.71145276	145.87514173	149.03788313
152.19973708	155.36075869	158.52099853	161.68050318	164.83931554
167.99747522	171.15501884	174.31198034	177.46839118	180.62428058
20.14408270	25.49555871	29.67014737	33.50392932	37.16040124
40.70679543	44.17812771	47.59513048	50.97113292	54.31521595
57.63383980	60.93175771	64.21256287	67.47903264	70.73335426
73.97727786	77.21222318	80.43935593	83.65964336	86.87389573
90.08279749	93.28693128	96.48679650	99.68282391	102.87538721
106.06481235	109.25138502	112.43535674	115.61694991	118.79636197
121.97376881	125.14932768	128.32317965	131.49545161	134.66625806
137.83570260	141.00387919	144.17087326	147.33676268	150.50161857
153.66550603	156.82848475	159.99060959	163.15193105	166.31249570
169.47234655	172.63152339	175.79006310	178.94799989	182.10536555
21.18226963	26.60553392	30.82414780	34.69148395	38.37523617
41.94458620	45.43566830	48.86993352	52.26120709	55.61892969
58.94983212	62.25887691	65.54982274	68.82558023	72.08844556
75.34025903	78.58251593	81.81644575	85.04307016	88.26324612
91.47769848	94.68704502	97.89181585	101.09246876	104.28940132
107.48296063	110.67345123	113.86114149	117.04626891	120.22904452
123.40965647	126.58827313	129.76504562	132.94010999	136.11358910
139.28559418	142.45622617	145.62557693	148.79373022	151.96076258
155.12674413	158.29173920	161.45580693	164.61900179	167.78137405

170.94297014	174.10383307	177.26400267	180.42351594	183.58240728
22.21914648	27.71212684	31.97371522	35.87394150	39.58453089
43.17653646	46.68716642	50.13856248	53.54502718	56.91634787
60.25951651	63.57969798	66.88081125	70.16589626	73.43735505
76.69711577	79.94674733	83.18754160	86.42057347	89.64674568
92.86682276	96.08145700	99.29120879	102.49656256	105.69793942
108.89570741	112.09018972	115.28167140	118.47040496	121.65661491
124.84050162	128.02224448	131.20200466	134.37992735	137.55614375
140.73077270	143.90392215	147.07569037	150.24616703	153.41543413
156.58356680	159.75063402	162.91669926	166.08182098	169.24605316
172.40944570	175.57204480	178.73389334	181.89503109	185.05549510

PARAMETER Z IN FORMULA FOR LARGE N: (Z(I),I=1,76) FOR 0.0<-ZETA<7.5
PARAMETER Z IN FORMULA FOR LARGE N: (Z(I),I=77,96) FOR 0.00<XI<0.38

1000000000.	1081258212.	1166283624.	1255057958.	1347557490.
1443753879.	1543614917.	1647105219.	1754186836.	1864819802.
1978962618.	2096572665.	2217606570.	2342020514.	2469770499.
2600812563.	2735102973.	2872598376.	3013255919.	3157033362.
3303889146.	3453782466.	3606673313.	3762522511.	3921291740.
4082943554.	4247441386.	4414749549.	4584833234.	4757658503.
4933192274.	5111402309.	5292257200.	5475726346.	5661779940.
5850388945.	6041525074.	6235160771.	6431269189.	6629824168.
6830800216.	7034172486.	7239916758.	7448009419.	7658427441.
7871148366.	8086150282.	8303411811.	8522912087.	8744630743.
8968547891.	9194644107.	9422900419.	9653298289.	9885819599.
10120446637.	10357162089.	10595949017.	10836790856.	11079671396.
11324574773.	11571485459.	11820388250.	12071268256.	12324110892.
12578901868.	12835627181.	13094273105.	13354826183.	13617273222.
13881601278.	14147797657.	14415849903.	14685745791.	14957473322.
15231020716.	1570796327.	1570790327.	1570748334.	1570634409.
1570412789.	1570048089.	1569505577.	1568751527.	1567753625.
1566481429.	1564906855.	1563004678.	1560753018.	1558133782.
1555133056.	1551741393.	1547954010.	1543770858.	1539196572.
1534240288.				

SECOND DIFFERENCES FOR Z: (ZD2(I),I=1,96)

3780492.	3768362.	3749920.	3726047.	3697565.
3665231.	3629733.	3591683.	3551630.	3510052.
3467368.	3423938.	3380072.	3336031.	3292037.
3248275.	3204898.	3162030.	3119771.	3078203.
3037388.	2997373.	2958193.	2919871.	2882423.
2845855.	2810170.	2775363.	2741425.	2708346.
2676112.	2644706.	2614110.	2584306.	2555273.
2526990.	2499438.	2472594.	2446438.	2420950.
2396107.	2371891.	2348281.	2325257.	2302801.
2280894.	2259518.	2238656.	2218291.	2198406.

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 2178986. 2160016. 2141480. 2123365. 2105657.
 2088342. 2071409. 2054845. 2038637. 2022776.
 2007249. 1992047. 1977159. 1962576. 1948288.
 1934286. 1920562. 1907107. 1893913. 1880973.
 1868279. 1855824. 1843601. 1831603. 1819824.
 1808258.       1.   -36000.  -71961. -107761.
 -143199. -177994. -211796. -244195. -274728.
 -302904. -328219. -350184. -368346. -382318.
 -391799. -396596. -396635. -391968. -382776.
 -369355.
FOURTH DIFFERENCES FOR Z: (ZD4(I),I=1,96)
-7.-6.-5.-5.-4.
-3.-3.-2.-2.-1.
-1. 0. 0. 0. 0.
 0. 1. 1. 1. 1.
 1. 1. 1. 1. 1.
 1. 1. 1. 1. 1.
 1. 1. 1. 1. 1.
 1. 1. 1. 1. 1.
 1. 1. 1. 1. 1.
 1. 1. 0. 0. 0.
 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.
 1. 1. 1. 2. 2.
 3. 3. 4. 4. 5.
 5. 5. 5. 5. 4.
 4.
PARAMETER P1 IN FORMULA FOR LARGE N: (P1(I),I=1,96)
 142857. 142398. 141553. 140368. 138884.
 137145. 135189. 133053. 130770. 128371.
 125883. 123329. 120731. 118107. 115474.
 112844. 110229. 107640. 105084. 102567.
 100095.  97672.  95302.  92986.  90728.
  88527.  86384.  84300.  82274.  80307.
  78397.  76543.  74745.  73001.  71310.
  69671.  68082.  66542.  65050.  63604.
  62202.  60843.  59527.  58250.  57013.
  55814.  54651.  53523.  52429.  51367.
  50338.  49338.  48368.  47427.  46513.
  45625.  44762.  43924.  43110.  42319.
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  41550.  40802.  40074.  39366.  38678.
  38008.  37355.  36720.  36102.  35499.
  34913.  34341.  33784.  33241.  32711.
  32195.      0.      7.     53.    180.
    426.    829.   1427.   2254.   3341.
   4714.   6395.   8398.  10732.  13395.
  16380.  19671   23246.  27074.  31122.
  35350.
SECOND DIFFERENCES FOR PARAMETER P1: (P1D2(I), I=1,96)
-429.-385.-341.-297.-256.
-216.-180.-146.-116. -89.
 -65. -44. -25.  -9.   4.
  16.  25.  33.  40.  45.
  49.  53.  55.  57.  58.
  58.  59.  59.  58.  57.
  57.  55.  54.  53.  52.
  50.  49.  48.  46.  45.
  43.  42.  40.  39.  38.
  36.  35.  34.  33.  32.
  30.  29.  28.  27.  26.
  25.  25.  24.  23.  22.
  21.  21.  20.  19.  18.
  18.  17.  17.  16.  16.
  15.  15.  14.  14.  13.
  13.   0.  40.  80. 119.
 158. 195. 230. 261. 287.
 309. 324. 331. 331. 323.
 308. 285. 255. 220. 181.
 139.
PARAMETER P2 IN FORMULA FOR LARGE N: (P2(I), I=1,76)
-119.-113.-108.-102. -96.
 -90. -84. -78. -73. -67.
 -62. -57. -53. -49. -45.
 -41. -38. -35. -32. -30.
 -27. -25. -23. -21. -20.
 -18. -17. -15. -14. -13.
 -12. -11. -10. -10.  -9.
  -8.  -8.  -7.  -7.  -6.
  -6.  -5.  -5.  -5.  -4.
  -4.  -4.  -4.  -3.  -3.
  -3.  -3.  -3.  -2.  -2.
  -2.  -2.  -2.  -2.  -2.
  -2.  -2.  -1.  -1.  -1.
  -1.  -1.  -1.  -1.  -1.
```

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 -1.  -1.  -1.  -1.  -1.
 -1.
PARAMETER Q1 IN FORMULA FOR LARGE N: (Q1(I), I=1,97)
-1259921.-1298628.-1334723.-1368192.-1399054.
-1427356.-1453166.-1476573.-1497675.-1516583.
-1533413.-1533413.-1407530.-1301097.-1209715.
-1130245.-1060379. -998386. -942938. -892999.
 -847749. -806530. -768803. -734130. -702142.
 -672532. -645039. -619441. -595548. -573193.
 -552234. -532545. -514015. -496546. -480051.
 -464454. -449685. -435682. -422389. -409755.
 -397735. -386287. -375372. -364956. -355008.
 -345499. -336400. -327689. -319341. -311337.
 -303657. -296282. -289195. -282382. -275828.
 -269519. -263443. -257588. -251942. -246496.
 -241240. -236164. -231261. -226522. -221940.
 -217508. -213218. -209065. -205043. -201146.
 -197368. -193705. -190152. -186704. -183358.
 -180108. -176952.       0.     -33.     -67.
    -899.   -2130.   -4153.   -7161.  -11335.
  -16848.  -23860.  -32514.  -42931.  -55212.
  -69432.  -85641. -103861. -124088. -146296.
 -170434. -196434.
SECOND DIFFERENCES FOR PARAMETER Q1: (Q1D2(I), I=1,97)
  2569.  2620.  2632.  2612.  2564.
  2494.  2407.  2306.  2195.  2079.
  1959.-25249.-19107.-14831.-11763.
 -9502. -7800. -6492. -5471. -4660.
 -4008. -3477. -3039. -2675. -2369.
 -2110. -1889. -1699. -1535. -1392.
 -1267. -1157. -1059.  -973.  -896.
  -827.  -765.  -709.  -658.  -613.
  -571.  -533.  -499.  -467.  -438.
  -411.  -386.  -364.  -343.  -323.
  -305.  -289.  -273.  -259.  -245.
  -233.  -221.  -210.  -199.  -190.
  -181.  -172.  -164.  -157.  -150.
  -143.  -137.  -131.  -125.  -120.
  -115.  -110.  -105.  -101.   -97.
   -93.   -90.     0.  -200.  -400.
  -598.  -794.  -985. -1169. -1342.
 -1502. -1645. -1768. -1868. -1944.
 -1993. -2016. -2012. -1984. -1934.
 -1865. -1780.
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FOURTH DIFFERENCES FOR PARAMETER Q1: (Q1D4(I), I=1,17)
 0. 0. 0. 0. 0.
 0. 0. 0. 0. 0.
 0.-3.-2.-1.-1.
-1. 0.
PARAMETER Q2 IN FORMULA FOR LARGE N: (Q2(I), I=1,50)
-10000. -9885. -9749. -9590. -9409
 -9205. -8979. -8734. -8471. -8193.
 -7903.  -790.  -571.  -422.  -318.
  -243.  -189.  -148.  -117.   -93.
   -75.   -61.   -50.   -41.   -33.
   -28.   -23.   -19.   -16.   -13.
   -11.    -9.    -8.    -7.    -6.
    -5.    -4.    -3.    -3.    -2.
    -2.    -2.    -1.    -1.    -1.
    -1.    -1.    -1.    -1.     0.
SECOND DIFFERENCES FOR PARAMETER Q2: (Q2D2(I), I=1,30)
  18.  21.  22.  23.  23.
  22.  20.  18.  15.  12.
   9.-108. -67. -43. -29.
 -19. -14. -10.  -7.  -5.
  -4.  -3.  -2.  -2.  -1.
  -1.  -1.  -1.  -1.   0.
PARAMETER Q3 IN FORMULA FOR LARGE N: (Q3(I), I=1,11)
-159.-156.-152.-148.-144.
-140.-137.-135.-133.-133.
-135.
NEGATIVE ZEROS OF THE AIRY FUNCTION: (A(S), S=1,50)
 -2.33810741 -4.08794944 -5.52055983 -6.78670809 -7.94413359
 -9.02265085-10.04017434-11.00852430-11.93601556-12.82877675
-13.69148904-14.52782995-15.34075514-16.13268516-16.90563400
-17.66130011-18.40113260-19.12638047-19.83812989-20.53733291
-21.22482994-21.90136760-22.56761292-23.22416500-23.87156446
-24.51030124-25.14082117-25.76353140-26.37880505-26.98698511
-27.58838781-28.18330550-28.77200917-29.35475056-29.93176412
-30.50326861-31.06946859-31.63055566-32.18670965-32.73809961
-33.28488468-33.82721495-34.36523213-34.89907025-35.42885619
-35.95471026-36.47674664-36.99507385-37.50979509-38.02100868
NEGATIVE ZEROS OF THE DERIVATIVE OF THE AIRY FUNCTION: (AP(S), S=1,50)
 -1.01879297 -3.24819758 -4.82009921 -6.16330736 -7.37217726
 -8.48848673 -9.53544905-10.52766040-11.47505663-12.38478837
-13.26221896-14.11150197-14.93593720-15.73820137-16.52050383
-17.28469505-18.03234462-18.76479844-19.48322166-20.18863151
-20.88192276-21.56388772-22.23523229-22.89658874-23.54852630
```

```
-24.19155971-24.82615643-25.45274256-26.07170794-26.68341033
-27.28817912-27.88631841-28.47810968-29.06381416-29.64367481
-30.21791812-30.78675565-31.35038538-31.90899296-32.46275275
-33.01182878-33.55637561-34.09653909-34.63245705-35.16425990
-35.69207120-36.21600815-36.73618208-37.25269882-37.76565910
```

2.2 The Output Data

The output data consist of the input data, some intermediate output data, and the final output data. The input data were described in Sections 2.1.1 and 2.1.2. One must read parts of Chapter 3 in order to interpret the intermediate output data. The meaning of the intermediate output data is evident from the description of the main program in Chapter 3. The final output data are described in Section 2.2.1.

2.2.1 Description of the Final Output Data

The final output data consist of E3A1PS(J), E3A1ZS(K), E3A2PS(J), E3A2ZS(K), BKAPLT(I), PTRAN(I), and PREFL(I) where $\{J = 1, 2, \cdots, \text{NPHI}\}$, $\{K = 1, 2, \cdots, \text{NZ}\}$, and $\{I = 1, 2, \cdots, \text{KAM}\}$. The E3A's are written at the end of DO loop 48. The variables BKAPLT, PTRAN, and PREFL are written at the end of the main program.

The E3A's are the magnitudes of the ϕ- and z-components of the normalized electric field along each of the two center lines in each of the two apertures. The E3A's are, as explained in the last paragraph of Section 2.1.1, written out only at those values of KA for which there is an integer J such that

$$\text{KE3(J)} = \text{KA}. \tag{2.58}$$

The E3A's are defined by

$$\mathrm{E3A1PS(J)} = \left| \frac{E_\phi^{(A1)}(\phi_\mathrm{J}^{(A1)}, 0)}{\left|E_{01}^{\mathrm{TMe}+}\right|_{\mathrm{rms}}} \right| \tag{2.59}$$

$$\mathrm{E3A1ZS(J)} = \left| \frac{E_z^{(A1)}(\pi, z_\mathrm{J}^{(A)})}{\left|E_{01}^{\mathrm{TMe}+}\right|_{\mathrm{rms}}} \right| \tag{2.60}$$

$$\mathrm{E3A2PS(J)} = \left| \frac{E_\phi^{(A2)}(\phi_\mathrm{J}^{(A2)}, 0)}{\left|E_{01}^{\mathrm{TMe}+}\right|_{\mathrm{rms}}} \right| \tag{2.61}$$

$$\mathrm{E3A2ZS(J)} = \left| \frac{E_z^{(A2)}(0, z_\mathrm{J}^{(A)})}{\left|E_{01}^{\mathrm{TMe}+}\right|_{\mathrm{rms}}} \right| . \tag{2.62}$$

Here, $E_\phi^{(A1)}(\phi_\mathrm{J}^{(A1)}, 0)$ is the ϕ-component of the electric field at $(\phi, z) = (\phi_\mathrm{J}^{(A1)}, 0)$ in the left-hand aperture, and $E_z^{(A1)}(\pi, z_\mathrm{J}^{(A)})$ is the z-component of the electric field at $(\phi, z) = (\pi, z_\mathrm{J}^{(A)})$ in the left-hand aperture. The coordinates ϕ and z are shown in Fig. 2. Now,

$$\phi_\mathrm{J}^{(A1)} = \pi + \left(-1 + 2\frac{\mathrm{J}-1}{\mathrm{NPHI}-1}\right)\phi_o \tag{2.63}$$

$$z_\mathrm{J}^{(A)} = \left(-1 + 2\frac{\mathrm{J}-1}{\mathrm{NZ}-1}\right)\frac{c}{2} . \tag{2.64}$$

Moreover, $E_\phi^{(A2)}(\phi_\mathrm{J}^{(A2)}, 0)$ is the ϕ-component of the electric field at $(\phi, z) = (\phi_\mathrm{J}^{(A2)}, 0)$ in the right-hand aperture, and $E_z^{(A2)}(0, z_\mathrm{J}^{(A)})$ is the z-component of the electric field at $(\phi, z) = (0, z_\mathrm{J}^{(A)})$ in the right-hand aperture. Here,

$$\phi_\mathrm{J}^{(A2)} = \left(-1 + 2\frac{\mathrm{J}-1}{\mathrm{NPHI}-1}\right)\phi_o \tag{2.65}$$

and $z_\mathrm{J}^{(A)}$ is given by (2.64). In (2.59)–(2.62), $\left|E_{01}^{\mathrm{TMe}+}\right|_{\mathrm{rms}}$ is the square root of the average value of the square of the magnitude of the tangential electric field of the z-traveling TM_{01}^e wave taken over one of the apertures.†

†This average value is the same over both apertures. The z-traveling TM_{01}^e wave is the incident wave in the circular waveguide.

The quantity PTRAN(I) is the ratio of the time-average power transmitted into the rectangular waveguides to the time-average power of the z-traveling TM_{01}^{e} wave in the circular waveguide when

$$ka = \mathrm{BKAPLT(I)} \tag{2.66}$$

where

$$\mathrm{BKAPLT(I)} = \mathrm{BKA0} + (\mathrm{I} - 1) * \mathrm{DBKA}. \tag{2.67}$$

Similarly, PREFL(I) is the ratio of the time-average power reflected in the circular waveguide to the time-average power of the z-traveling TM_{01}^{e} wave in the circular waveguide when ka is given by (2.66). Because the medium in the waveguides is assumed to be lossless,

$$\mathrm{PTRAN(I)} + \mathrm{PREFL(I)} = 1. \tag{2.68}$$

2.2.2 Sample Output Data

When the computer program was run with the input data listed in Section 2.1.3, the output data listed below were written in the file OUT.DAT by statements in the main program.

Listing of the sample output data written in the file OUT.DAT by statements in the main program

```
B,C,L1,L2,L3,BKM,XM,ZL1,ZL2
 0.1100000D+01 0.5000000D+00 0.4000000D+02 0.4000000D+02
 0.1838694D+01 0.1500000D+02 0.4000000D+02 0.1000000D+01
 0.0000000D+00 0.1000000D+01 0.0000000D+00
KAM=    1, BKA0= 0.2950000D+01, DBKA= 0.0000000D+00
KE3M=    1, NPHI=  81, NZ=  21
KE3
   1
   5    5    2
KTM=    5, KTE=   11, K1=   16
BKB= 0.3245000E+01
YREC
 0.4727867E+00 0.0000000E+00 0.3704829E+00 0.0000000E+00
 0.2890127E+00 0.0000000E+00 0.2322885E+00 0.0000000E+00
 0.2351603E+00 0.0000000E+00 0.0000000E+00-0.2504353E+00
```

```
-0.1658050E+01 0.0000000E+00-0.2726819E+01 0.0000000E+00
-0.3741192E+01 0.0000000E+00-0.1880544E+01 0.0000000E+00
-0.2115119E+01 0.0000000E+00-0.2699180E+01 0.0000000E+00
-0.3460056E+01 0.0000000E+00-0.4304992E+01 0.0000000E+00
-0.4140747E+01 0.0000000E+00-0.4252419E+01 0.0000000E+00
 0.4727867E+00 0.0000000E+00 0.3704829E+00 0.0000000E+00
 0.2890127E+00 0.0000000E+00 0.2322885E+00 0.0000000E+00
 0.2351603E+00 0.0000000E+00 0.0000000E+00-0.2504353E+00
-0.1658050E+01 0.0000000E+00-0.2726819E+01 0.0000000E+00
-0.3741192E+01 0.0000000E+00-0.1880544E+01 0.0000000E+00
-0.2115119E+01 0.0000000E+00-0.2699180E+01 0.0000000E+00
-0.3460056E+01 0.0000000E+00-0.4304992E+01 0.0000000E+00
-0.4140747E+01 0.0000000E+00-0.4252419E+01 0.0000000E+00
TI
 0.0000000E+00 0.8683748E-08 0.0000000E+00 0.0000000E+00
 0.0000000E+00 0.1880288E-08 0.0000000E+00 0.0000000E+00
 0.0000000E+00-0.2009645E-01 0.0000000E+00 0.7737053E+00
 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.2579018E+00
 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 0.0000000E+00 0.3947158E-08 0.0000000E+00 0.0000000E+00
 0.0000000E+00 0.2564029E-08 0.0000000E+00 0.0000000E+00
 0.0000000E+00 0.0000000E+00 0.0000000E+00-0.4567374E-02
 0.0000000E+00 0.8683748E-08 0.0000000E+00 0.0000000E+00
 0.0000000E+00 0.1880288E-08 0.0000000E+00 0.0000000E+00
 0.0000000E+00-0.2009645E-01 0.0000000E+00 0.7737053E+00
 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.2579018E+00
 0.0000000E+00 0.0000000E+00 0.0000000E+00 0.0000000E+00
 0.0000000E+00 0.3947158E-08 0.0000000E+00 0.0000000E+00
 0.0000000E+00 0.2564029E-08 0.0000000E+00 0.0000000E+00
 0.0000000E+00 0.0000000E+00 0.0000000E+00-0.4567374E-02
V
-0.6700145E-01-0.6189583E-02 0.4555167E-16-0.1384864E-17
-0.2633363E-01-0.2438369E-02 0.7368085E-16 0.3595178E-17
-0.1930133E+00-0.3291131E-01-0.9979596E+00-0.8676731E-01
-0.5602431E-17-0.9498243E-18-0.3494251E-01-0.2162649E-01
-0.7301090E-18-0.2095366E-18-0.3495097E-17 0.1135324E-16
 0.1183619E-01 0.1109780E-02-0.1387413E-16-0.6655160E-18
 0.3481302E-02 0.3235679E-03-0.1050499E-16-0.5683518E-18
-0.2474749E-16-0.3544028E-17-0.3542653E-01-0.2582735E-02
-0.6700145E-01-0.6189589E-02 0.4555165E-16-0.1384874E-17
-0.2633362E-01-0.2438373E-02 0.7368087E-16 0.3595155E-17
-0.1930133E+00-0.3291131E-01-0.9979594E+00-0.8676729E-01
-0.5602437E-17-0.9498248E-18-0.3494251E-01-0.2162649E-01
-0.7301109E-18-0.2095369E-18-0.3495087E-17 0.1135325E-16
```

```
 0.1183619E-01 0.1109779E-02-0.1387413E-16-0.6655197E-18
 0.3481302E-02 0.3235681E-03-0.1050499E-16-0.5683519E-18
-0.2474748E-16-0.3544027E-17-0.3542653E-01-0.2582736E-02
C1OUT=-0.6562232E+00-0.5705513E-01, C1IN= 0.0000000E+00 0.0000000E+00
C2OUT=-0.6562231E+00-0.5705512E-01, C2IN= 0.0000000E+00 0.0000000E+00
CTME= 0.3417053E+00 0.1243756E+00, CTEE= 0.1253118E-15 0.1798084E-21
CTEO= 0.3527384E-07 0.4016329E-08
C1OUTS= 0.4338841E+00, C1INS= 0.0000000E+00, C2OUTS= 0.4338840E+00
C2INS= 0.0000000E+00, PT= 0.8677682E+00
CTMES= 0.1322318E+00 ,CTMMS= 0.1815642E+01
CTEES= 0.1570306E-31 CTEOS= 0.1260375E-14
PR= 0.1322318E+00 ,PRM= 0.1815642E+01
PTOTAL= 0.1000000E+01
PTA= 0.8677680E+00, PRMA= 0.1815643E+01
PHI2
 0.0000000E+00 0.3926991E-01 0.7853982E-01 0.1178097E+00 0.1570796E+00
 0.1963495E+00 0.2356195E+00 0.2748893E+00 0.3141593E+00 0.3534292E+00
 0.3926991E+00 0.4319690E+00 0.4712389E+00 0.5105088E+00 0.5497787E+00
 0.5890486E+00 0.6283185E+00 0.6675884E+00 0.7068583E+00 0.7461283E+00
 0.7853982E+00 0.8246680E+00 0.8639380E+00 0.9032079E+00 0.9424778E+00
 0.9817477E+00 0.1021018E+01 0.1060287E+01 0.1099557E+01 0.1138827E+01
 0.1178097E+01 0.1217367E+01 0.1256637E+01 0.1295907E+01 0.1335177E+01
 0.1374447E+01 0.1413717E+01 0.1452987E+01 0.1492257E+01 0.1531526E+01
 0.1570796E+01 0.1610066E+01 0.1649336E+01 0.1688606E+01 0.1727876E+01
 0.1767146E+01 0.1806416E+01 0.1845686E+01 0.1884956E+01 0.1924225E+01
 0.1963495E+01 0.2002765E+01 0.2042035E+01 0.2081305E+01 0.2120575E+01
 0.2159845E+01 0.2199115E+01 0.2238385E+01 0.2277655E+01 0.2316925E+01
 0.2356194E+01 0.2395464E+01 0.2434734E+01 0.2474004E+01 0.2513274E+01
 0.2552544E+01 0.2591814E+01 0.2631084E+01 0.2670354E+01 0.2709624E+01
 0.2748893E+01 0.2788163E+01 0.2827433E+01 0.2866703E+01 0.2905973E+01
 0.2945243E+01 0.2984513E+01 0.3023783E+01 0.3063053E+01 0.3102323E+01
 0.3141593E+01
Z
 0.0000000E+00 0.1570796E+00 0.3141593E+00 0.4712389E+00 0.6283185E+00
 0.7853982E+00 0.9424778E+00 0.1099557E+01 0.1256637E+01 0.1413717E+01
 0.1570796E+01 0.1727876E+01 0.1884956E+01 0.2042035E+01 0.2199115E+01
 0.2356194E+01 0.2513274E+01 0.2670354E+01 0.2827433E+01 0.2984513E+01
 0.3141593E+01
E3A1PS
 0.4837097E+00 0.4822135E+00 0.4777427E+00 0.4703507E+00 0.4601257E+00
 0.4471895E+00 0.4316956E+00 0.4138279E+00 0.3937970E+00 0.3718385E+00
 0.3482087E+00 0.3231825E+00 0.2970478E+00 0.2701041E+00 0.2426554E+00
 0.2150089E+00 0.1874691E+00 0.1603345E+00 0.1338933E+00 0.1084194E+00
 0.8416927E-01 0.6137794E-01 0.4025627E-01 0.2098808E-01 0.3727845E-02
```

```
 0.1140163E-01 0.2430998E-01 0.3494114E-01 0.4327350E-01 0.4932024E-01
 0.5312867E-01 0.5477954E-01 0.5438583E-01 0.5209066E-01 0.4806545E-01
 0.4250704E-01 0.3563496E-01 0.2768797E-01 0.1892070E-01 0.9599797E-02
 0.1099901E-07 0.9599783E-02 0.1892069E-01 0.2768796E-01 0.3563494E-01
 0.4250703E-01 0.4806544E-01 0.5209063E-01 0.5438579E-01 0.5477954E-01
 0.5312865E-01 0.4932025E-01 0.4327353E-01 0.3494110E-01 0.2430998E-01
 0.1140160E-01 0.3727860E-02 0.2098808E-01 0.4025627E-01 0.6137794E-01
 0.8416927E-01 0.1084194E+00 0.1338933E+00 0.1603345E+00 0.1874691E+00
 0.2150088E+00 0.2426554E+00 0.2701040E+00 0.2970479E+00 0.3231825E+00
 0.3482088E+00 0.3718385E+00 0.3937971E+00 0.4138279E+00 0.4316956E+00
 0.4471894E+00 0.4601257E+00 0.4703507E+00 0.4777426E+00 0.4822135E+00
 0.4837097E+00
E3A1ZS
 0.7626179E+01 0.7541701E+01 0.7296243E+01 0.6912951E+01 0.6427896E+01
 0.5886560E+01 0.5339398E+01 0.4836897E+01 0.4424638E+01 0.4138892E+01
 0.4003272E+01 0.4026743E+01 0.4203009E+01 0.4511133E+01 0.4917455E+01
 0.5378818E+01 0.5846881E+01 0.6273001E+01 0.6613139E+01 0.6832278E+01
 0.6907912E+01
E3A2PS
 0.4837097E+00 0.4822134E+00 0.4777426E+00 0.4703507E+00 0.4601257E+00
 0.4471894E+00 0.4316956E+00 0.4138279E+00 0.3937971E+00 0.3718385E+00
 0.3482088E+00 0.3231825E+00 0.2970479E+00 0.2701040E+00 0.2426554E+00
 0.2150088E+00 0.1874691E+00 0.1603345E+00 0.1338933E+00 0.1084194E+00
 0.8416928E-01 0.6137797E-01 0.4025628E-01 0.2098811E-01 0.3727888E-02
 0.1140158E-01 0.2430994E-01 0.3494107E-01 0.4327351E-01 0.4932023E-01
 0.5312863E-01 0.5477951E-01 0.5438576E-01 0.5209060E-01 0.4806541E-01
 0.4250703E-01 0.3563493E-01 0.2768795E-01 0.1892068E-01 0.9599781E-02
 0.1099901E-07 0.9599794E-02 0.1892070E-01 0.2768797E-01 0.3563494E-01
 0.4250704E-01 0.4806542E-01 0.5209064E-01 0.5438580E-01 0.5477954E-01
 0.5312865E-01 0.4932021E-01 0.4327348E-01 0.3494111E-01 0.2430994E-01
 0.1140161E-01 0.3727873E-02 0.2098811E-01 0.4025628E-01 0.6137797E-01
 0.8416928E-01 0.1084194E+00 0.1338933E+00 0.1603345E+00 0.1874691E+00
 0.2150089E+00 0.2426554E+00 0.2701041E+00 0.2970478E+00 0.3231825E+00
 0.3482087E+00 0.3718385E+00 0.3937970E+00 0.4138279E+00 0.4316956E+00
 0.4471894E+00 0.4601257E+00 0.4703507E+00 0.4777426E+00 0.4822134E+00
 0.4837097E+00
E3A2ZS
 0.7626177E+01 0.7541700E+01 0.7296242E+01 0.6912949E+01 0.6427894E+01
 0.5886559E+01 0.5339396E+01 0.4836895E+01 0.4424636E+01 0.4138891E+01
 0.4003270E+01 0.4026742E+01 0.4203007E+01 0.4511131E+01 0.4917453E+01
 0.5378817E+01 0.5846880E+01 0.6272999E+01 0.6613138E+01 0.6832277E+01
 0.6907911E+01
BKAPLT
 0.2950000E+01
```

```
PTRAN
 0.8677682E+00
PREFL
 0.1322318E+00
```

Discussion of the above sample output data

In the above sample output data, the values of E3A2PS should be the same as those of E3A1PS, and the values of E3A2ZS should be the same as those of E3A1ZS. The occasional differences of one or two units in the seventh significant figure are due to roundoff error.

The value of PTRAN(1) in the sample output data is the same as that of P_t at $ka = 2.95$ in Fig. 8.4 of [2]. This value is the same as that of P_t at $L_3/\lambda_{01}^{\rm TM} = 0.5$ in Fig. 8.6 of [2].

To see if BKM = 15 and XM = 40 of (2.52) and (2.53) are large enough to give accurate results for the time-average transmitted and reflected powers and the tangential electric field in the apertures, we ran the computer program with the input data changed so that BKM = 33 and XM = 100. The results for the output variables E3A2PS, E3A2ZS, BKAPLT, PTRAN and PREFL are shown below.

```
E3A2PS
 0.6727512E+00 0.6614469E+00 0.6284841E+00 0.5766137E+00 0.5101027E+00
 0.4342954E+00 0.3550694E+00 0.2782472E+00 0.2090295E+00 0.1515097E+00
 0.1083235E+00 0.8046831E-01 0.6730618E-01 0.6674646E-01 0.7558026E-01
 0.8992325E-01 0.1057121E+00 0.1191946E+00 0.1273567E+00 0.1282371E+00
 0.1210977E+00 0.1064339E+00 0.8583125E-01 0.6168983E-01 0.3685839E-01
 0.1422592E-01 0.3676301E-02 0.1500904E-01 0.1885962E-01 0.1534288E-01
 0.5564338E-02 0.8549457E-02 0.2452678E-01 0.3969521E-01 0.5154726E-01
 0.5807864E-01 0.5805428E-01 0.5116809E-01 0.3807576E-01 0.2029656E-01
 0.4981872E-08 0.2029661E-01 0.3807564E-01 0.5116813E-01 0.5805422E-01
 0.5807864E-01 0.5154727E-01 0.3969515E-01 0.2452665E-01 0.8549399E-02
 0.5564411E-02 0.1534297E-01 0.1885962E-01 0.1500908E-01 0.3676274E-02
 0.1422588E-01 0.3685847E-01 0.6168973E-01 0.8583125E-01 0.1064339E+00
 0.1210976E+00 0.1282370E+00 0.1273567E+00 0.1191946E+00 0.1057119E+00
 0.8992319E-01 0.7558020E-01 0.6674645E-01 0.6730618E-01 0.8046830E-01
 0.1083234E+00 0.1515096E+00 0.2090295E+00 0.2782472E+00 0.3550694E+00
 0.4342955E+00 0.5101028E+00 0.5766138E+00 0.6284842E+00 0.6614470E+00
 0.6727512E+00
E3A2ZS
 0.8836988E+01 0.8538660E+01 0.7732039E+01 0.6651182E+01 0.5594166E+01
 0.4820989E+01 0.4468800E+01 0.4515595E+01 0.4803845E+01 0.5112722E+01
 0.5250993E+01 0.5134759E+01 0.4819659E+01 0.4475413E+01 0.4313334E+01
```

```
 0.4495948E+01 0.5064260E+01 0.5910543E+01 0.6808496E+01 0.7490894E+01
 0.7745238E+01
BKAPLT
 0.2950000E+01
PTRAN
 0.8663139E+00
PREFL
 0.1336857E+00
```

Note that the above values of E3A2PS and E3A2ZS are considerably different from those computed with BKM = 15 and XM = 40. However, the above values of PTRAN and PREFL are quite close to those computed with BKM = 15 and XM = 40.† We surmise that the values of PTRAN and PREFL are accurate but that the values of E3A2PS and E3A2ZS are not.

The curves of Figs. 8.7 to 8.13 of [2] are labeled wrong. These curves are plots of the squares of the indicated normalized aperture fields rather than the normalized aperture fields themselves. For instance, the curve of Fig. 8.7a of [2] is a plot of $|E_\phi^{(A2)}/|\underline{E}_{01}^{\mathrm{TMe}+}|_{\mathrm{rms}}|^2$ rather than that of $|E_\phi^{(A2)}|/|\underline{E}_{01}^{\mathrm{TMe}+}|_{\mathrm{rms}}$. The curve of the squares of the values of E3A2PS computed with BKA = 33 and XM = 100 coincides with the curve in Fig. 8.10(a) of [2]. The curve of the squares of the values of E3A2ZS computed with BKM = 33 and XM = 100 coincides with the curve in Fig. 8.10(b) of [2].

Notice that the values of $|E_\phi^{(A2)}|/|\underline{E}_{01}^{\mathrm{TMe}+}|_{\mathrm{rms}}$ at $(\phi, z) = (\pm\phi_o, 0)$ and those of $|E_z^{(A2)}|/|\underline{E}_{01}^{\mathrm{TMe}+}|_{\mathrm{rms}}$ at $(\phi, z) = (0, \pm c/2)$ increased when (BKM, XM) was increased from (15, 40) to (33, 100). Theory predicts that the component of electric field perpendicular to any edge of the aperture becomes infinite as this edge is approached (see Section 1.11.2 of [5]). Therefore, the computed values of $|E_\phi^{(A2)}|/|\underline{E}_{01}^{\mathrm{TMe}+}|_{\mathrm{rms}}$ at $(\phi, z) = (\pm\phi_o, 0)$ and those of $|E_z^{(A2)}|/|\underline{E}_{01}^{\mathrm{TMe}+}|_{\mathrm{rms}}$ at $(\phi, z) = (0, \pm c/2)$ cannot be accurate; they would probably increase more and more as BKM and XM were made larger and larger.

Sample output written in the file BESOUT by statements in the subroutine BESIN

All except the last line of the output data written in the file BESOUT by statements in the subroutine BESIN consist of data that were read in by

†Notice that the sum of the values of PTRAN and PREFL computed with BKM = 33 and XM = 100 is 0.9999997 rather that 1.0. This discrepancy can be attributed to roundoff error becuase not all calculations were done in double precision.

statements in the subroutine BES. These data are not listed in the present report. They were written out merely to verify that the subroutine BESIN received its input data properly.

The last line of the output data written in the file BESOUT by statements in the subroutine BESIN is

```
A(50)=-38.02100868, AP(50)=-37.76565910.
```

In the above line of output data, A(50) is the computed value of the 50^{th} negative root of the Airy function Ai, and AP(50) is the computed value of the 50^{th} negative root of the derivative of Ai. Note that all ten significant figures of these computed values are exactly the same as those of the tabulated values in the listing of the second module of sample input data in Section 2.1.3.

2.3 Minimum Allocations

The minimum storage space that must be allocated to some arrays in the computer program depends on the values of the input variables B, C, BKM, KAM, KE3M, NPHI, and NZ.

Minimum allocations in the main program are given by

```
MM(NMAX), BMN(KTE), BMN2(KTE), PH1(2*PMAX), PH2(2*PMAX),
PH3(2*PMAX), PH4(2*PMAX), D3(NMAX), G4(NMAX), DTM(NMAX),
DTE(NMAX), E3A1P(NPHI), E3A1Z(NZ), E3A2P(NPHI), E3A2Z(NZ),
Y(K2*K2), TI(K2), V(K2), CVTME(K2), CVTEE(K2), CVTEO(K2),
YREC(K2), GTM(NMAX), GTE(NMAX), TMP(NMAX), TMM(NMAX),
TEP(NMAX), TEM(NMAX), DQTM(NMAX), DQTE(NMAX), PHI1(NPHI),
PHI2(NPHI), Z(NZ), PTRAN(KAM), PREFL(KAM), BKAPLT(KAM),
SINP(PMAX), SINQ(PMAX), E3A1PS(NPHI), E3A2PS(NPHI),
E3A1ZS(NZ), E3A2ZS(NZ), IPS(K2), and KE3(KE3M).
```

Here,

$$\mathrm{NMAX} = 1 + \left\{ \begin{array}{c} \text{the maximum value of } n \text{ such that} \\ (n\pi)(\mathrm{B/C}) \leq \mathrm{BKM} \end{array} \right\} \tag{2.69}$$

and

$$\mathrm{KTE} = -1 + \left\{ \begin{array}{l} \text{the number of combinations of nonnegative} \\ \text{integers } m \text{ and n such that} \\ \qquad\qquad \sqrt{(m\pi)^2 + (n\pi(\mathrm{B/C}))^2} \leq \mathrm{BKM} \end{array} \right\}. \tag{2.70}$$

Furthermore,

$$\text{PMAX} = 1 + \left\{ \begin{array}{c} \text{the maximum value of } p \text{ such that} \\ p\pi \leq \text{BKM} \end{array} \right\}. \tag{2.71}$$

Moreover,

$$\text{K2} = 2 * (\text{KTM} + \text{KTE}) \tag{2.72}$$

where KTE is given by (2.70) and

$$\text{KTM} = \left\{ \begin{array}{l} \text{the number of combinations of} \\ \text{positive integers } m \text{ and } n \text{ such that} \\ \qquad \sqrt{(m\pi)^2 + (n\pi(\text{B}/\text{C}))^2} \leq \text{BKM} \end{array} \right\}. \tag{2.73}$$

Minimum allocations in the subroutine MODES are given by

`MM(NMAX), BMN(KTE), and BMN2(KTE).`

Minimum allocations in the subroutine PHI are given by

`PH1(2*PMAX), PH2(2*PMAX), PH3(2*PMAX), and PH4(2*PMAX).`

Minimum allocations in the subroutine DGN are given by

`D3(NMAX), G4(NMAX), D(NMAX), CP(NMAX), CM(NMAX), DQ(NMAX), and G(NMAX).`

Minimum allocations in the subroutine DECOMP are given by

`UL(K2*K2), SCL(K2), and IPS(K2).`

Minimum allocations in the subroutine SOLVE are given by

`UL(K2*K2), B(K2), X(K2), and IPS(K2).`

A blank or labeled common block that is used in two or more program segments must be defined exactly the same in each of these program segments.† Therefore, any dimensioned variable in a common block that is used in two

†Here, a program segment is either the main program or one of the subprograms.

or more program segments must have the same allocation in each of these program segments.

The computer program was written assuming that

$$x'_{1,200} > \text{XM} \tag{2.74}$$

where $x'_{1,200}$ is the 200^{th} root of J'_1. See (2.11). If (2.74) is not true, then the upper limits of the indices of DO loops 14 and 15 in the subroutine BES must be increased from 200 to an integer I at least so large that

$$x'_{1,\text{I}} > \text{XM}. \tag{2.75}$$

With this increase, $\{$A(I) and AP(I) for I $= 201, 202, \cdots,$ I$\}$ might be needed for use in DO loop 15 of the subroutine BES. These additional A's and AP's can be obtained by increasing the upper limit on the index of DO loop 25 in the subroutine BESIN from 200 to I. The "200" in the fifth statement in DO loop 19 of the main program must also be increased to I. Accompanying minimum allocations are given by

```
XJ(I) and XJP(I)
```

in the main program,

```
A(I) and AP(I)
```

in the subroutine BESIN,

```
A(I), AP(I), XJ(I), and XJP(I)
```

in the subroutine BES,

```
A(I) and AP(I)
```

in the subroutine INTERPOL, and

```
X(I)
```

in the subroutine DGN.

Chapter 3

The Main Program

Numerical values of variables in [2] are stored in variables in the main program. The main program is described by defining important computer program variables in terms of variables in [2].

3.1 Rectangular Waveguide Mode Cutoff Wavenumbers

After input data is read from the file IN.DAT and written in the file OUT.DAT, computer program variables PI, BC, PC, and BKM2 are defined as†

$$\mathrm{PI} = \pi \tag{3.1}$$
$$\mathrm{BC} = b/c \tag{3.2}$$
$$\mathrm{PC} = \pi b/c \tag{3.3}$$
$$\mathrm{BKM2} = (\mathrm{BKM})^2. \tag{3.4}$$

The statement "CALL MODES"‡ uses PI, PC, and BKM2 to calculate

$$\mathrm{BMN2(MTE)} = (k_{mn}b)^2, \begin{cases} m = 0, 1, 2, \cdots, \mathrm{MM}(n+1) - 1 \\ n = 0, 1, 2, \cdots, \mathrm{NMAX} - 1 \\ n + m \neq 0 \end{cases} \tag{3.5}$$

†See the listing of the main program in Section 3.11.

‡See Chapter 4 for a description and a listing of the subroutine MODES.

where

$$\mathrm{MTE} = m + \begin{cases} 0, & n = 0 \\ \sum_{l=1}^{n} \mathrm{MM}(l), & n > 0. \end{cases} \tag{3.6}$$

According to eq. (2.4) of [2],

$$(k_{mn}b)^2 = (m\pi)^2 + \left(\frac{n\pi b}{c}\right)^2. \tag{3.7}$$

The upper bounds $\{\mathrm{MM}(l),\ l = 1, 2, \cdots, \mathrm{NMAX}\}$ on $m + 1$ and the upper bound NMAX on $n + 1$ in (3.5) are chosen to limit the values of m and n exactly the same as they are limited by the constraint

$$(k_{mn}b)^2 \leq \mathrm{BKM2}. \tag{3.8}$$

The statement "CALL MODES" also performs the following operations. It makes available to the main program the values of NMAX and $\{\mathrm{MM}(l),\ l = 1, 2, \cdots, \mathrm{NMAX}\}$ in (3.5). It sets

$$\mathrm{BMN(MTE)} = k_{mn}b \tag{3.9}$$

where MTE, m, and n are the same as in (3.5). It sets KTE equal to the number of TE rectangular waveguide modes for which (3.8) holds.† It sets KTM equal to the number of TM rectangular waveguide modes for which (3.8) holds. Here, KTE and KTM are given by (4.13) and (4.14), respectively.

The statement following statement 102 sets K1 equal to the total number of TE and TM rectangular waveguide modes for which (3.8) holds:

$$\mathrm{K1} = \mathrm{KTM} + \mathrm{KTE}. \tag{3.10}$$

The four statements preceding statement 115 terminate execution if BKM is so small that there is no value of $(k_{mn}b)^2$ that satisfies (3.8).

3.2 The Parameter ka

The waveguide mode converter problem is solved for

$$ka = \mathrm{BKA0} + (\mathrm{KA} - 1) * \mathrm{DBKA} \tag{3.11}$$

†Here, KTE is the maximum of the values of MTE in (3.5).

inside DO loop 48. Before entry into DO loop 48, statement 115 puts in the common block labeled BESIN data that will be used by the subroutine BES to calculate roots of Bessel functions and roots of derivatives of Bessel functions. The three statements following statement 115 set

$$\mathrm{PI2} = 2\pi \tag{3.12}$$

$$\mathrm{PIBC} = \frac{\pi b}{c} \tag{3.13}$$

$$\mathrm{PI5} = \frac{\pi}{2}. \tag{3.14}$$

The variable KAE, which is set to 1 before entry into DO loop 48, appears in the list of FORTRAN statements before (2.36).

The first two statements in DO loop 48 set

$$\mathrm{BKA} = ka \tag{3.15}$$

$$\mathrm{BKA2} = (ka)^2 \tag{3.16}$$

where ka is given by (3.11). The eight statements before statement 96 terminate execution if ka does not satisfy eq. (8.9) of [2]. The four statements before statement 98 terminate execution if c is not less than b. Statement 98 sets

$$\mathrm{BKB} = kb. \tag{3.17}$$

The eight statements before statement 112 terminate execution if kb does not satisfy eq. (8.6) of [2].

3.3 The Admittance Matrices of the Rectangular Waveguides

The admittance matrices of the rectangular waveguides are Y^1 of (1.36) and Y^2 of (1.37). The eight Y submatrices on the right-hand sides of (1.36) and (1.37) are approximated by $\hat{Y}$ submatrices whose ij^{th} elements are given by eqs. (2.6), (2.12), (2.13), (2.16), (2.18), and (2.19) of [2]. All of these elements are zero except

$\{\hat{Y}_{ii}^{1,1\mathrm{TM},1\mathrm{TM}}, i = 1, 2, \cdots, \mathrm{KTM}\}$, $\{\hat{Y}_{ii}^{1,1\mathrm{TE},1\mathrm{TE}}, i = 1, 2, \cdots, \mathrm{KTE}\}$,
$\{\hat{Y}_{ii}^{2,2\mathrm{TM},2\mathrm{TM}}, i = 1, 2, \cdots, \mathrm{KTM}\}$, and $\{\hat{Y}_{ii}^{2,2\mathrm{TE},2\mathrm{TE}}, i = 1, 2, \cdots, \mathrm{KTE}\}$.

Nested DO loops 13 and 14 put

$\{-j\eta\hat{Y}_{ii}^{1,1\text{TM},1\text{TM}}, i = 1, 2, \cdots, \text{KTM}\}$, $\{-j\eta\hat{Y}_{ii}^{1,1\text{TE},1\text{TE}}, i = 1, 2, \cdots, \text{KTE}\}$,
$\{-j\eta\hat{Y}_{ii}^{2,2\text{TM},2\text{TM}}, i = 1, 2, \cdots, \text{KTM}\}$, and
$\{-j\eta\hat{Y}_{ii}^{2,2\text{TE},2\text{TE}}, i = 1, 2, \cdots, \text{KTE}\}$

in the order that they appear above in YREC(1) through YREC(2 * K1) where K1 is given by (3.10).

The 12 statements before DO loop 13 define variables that are used in DO loop 13. These statements set

$$\text{BKB2} = (kb)^2 \tag{3.18}$$

$$\text{BKR} = \frac{1}{kb} \tag{3.19}$$

$$\text{U} = j \tag{3.20}$$

$$\text{BKU} = -\frac{j}{kb} \tag{3.21}$$

$$\text{B5} = \sin\phi_o \tag{3.22}$$

$$\text{BX5} = \phi_o \tag{3.23}$$

$$\text{BX} = 2\phi_o \tag{3.24}$$

$$\text{XB} = \frac{x_o}{b} \tag{3.25}$$

$$\text{X1} = \frac{x_1}{b} \tag{3.26}$$

$$\text{X2} = \frac{x_2}{b} \tag{3.27}$$

$$\text{JTE} = 0 \tag{3.28}$$

$$\text{JTM} = 0 \tag{3.29}$$

where ϕ_o, x_o, x_1, and x_2 are given by, respectively, eqs. (2.9), (2.8), (2.7), and (2.17) of [2].

In nested DO loops 13 and 14,

$$\text{P} = p + 1 \tag{3.30}$$

$$\text{Q} = q + 1 \tag{3.31}$$

where p and q appear in eqs. (2.6), (2.12), (2.13), (2.16), (2.18), and (2.19) of [2]. In inner DO loop 14,

$$\mathrm{JTE} = \mathrm{P} - 1 + \begin{cases} 0, & \mathrm{Q} = 1 \\ \sum_{l=1}^{\mathrm{Q}-1} \mathrm{MM}(l), & \mathrm{Q} > 1. \end{cases} \tag{3.32}$$

The right-hand side of (3.32) is the right-hand side of (4.5) with M and N replaced by P and Q, respectively. The right-hand side of (3.32) is also the subscript i that appears in $\hat{Y}_{ii}^{1,1\mathrm{TE},1\mathrm{TE}}$. The variables JTE1 and JTE2 in DO loop 14 are such that $-j\eta\hat{Y}_{ii}^{1,1\mathrm{TE},1\mathrm{TE}}$ will be put in YREC(JTE1) and $-j\eta\hat{Y}_{ii}^{2,2\mathrm{TE},2\mathrm{TE}}$ will be put in YREC(JT2). In DO loop 14,

$$\mathrm{GAM2} = \gamma_{pq}^2 b^2 \tag{3.33}$$

where γ_{pq} is given by eq. (2.3) of [2].

If P = 2 and Q = 1 so that, according to (3.30) and (3.31), $p = 1$ and $q = 0$, then the ten statements following the branch statement

IF(P.NE.2.OR.Q.NE.1) GO TO 15

are executed. These statements set

$$\begin{aligned}
\mathrm{BET} &= \beta_{10} b && (3.34)\\
\mathrm{A1} &= \beta_{10} x_1 && (3.35)\\
\mathrm{CA} &= \cos \beta_{10} x_1 && (3.36)\\
\mathrm{SA} &= j \sin \beta_{10} x_1 && (3.37)\\
\mathrm{S1} &= -j\frac{\beta_{10}}{k} && (3.38)\\
\mathrm{YREC(JTE1)} &= -j\eta\hat{Y}_{ii}^{1,1\mathrm{TE},1\mathrm{TE}} && (3.39)\\
\mathrm{A2} &= \beta_{10} x_2 && (3.40)\\
\mathrm{CA} &= \cos \beta_{10} x_2 && (3.41)\\
\mathrm{SA} &= j \sin \beta_{10} x_2 && (3.42)\\
\mathrm{YREC(JTE2)} &= -j\eta\hat{Y}_{ii}^{2,2\mathrm{TE},2\mathrm{TE}} && (3.43)
\end{aligned}$$

where

$$\beta_{10} = \sqrt{k^2 - k_{10}^2}\ . \tag{3.44}$$

Furthermore, $-j\eta\hat{Y}_{ii}^{1,1\mathrm{TE},1\mathrm{TE}}$ is given by eq. (2.6) of [1] with δ_{ij} deleted and $-j\eta\hat{Y}_{ii}^{2,2\mathrm{TE},2\mathrm{TE}}$ is given by eq. (2.16) of [2] with δ_{ij} deleted.

If P $\neq$ 2 or if Q $\neq$ 1, then statement 15 and the three statements following it set

$$\mathrm{GAM} = \gamma_{pq}b \tag{3.45}$$

$$\mathrm{YTE} = -\frac{\gamma_{pq}}{k} \tag{3.46}$$

$$\mathrm{YREC(JTE1)} = -j\eta\hat{Y}_{ii}^{1,1\mathrm{TE},1\mathrm{TE}} \tag{3.47}$$

$$\mathrm{YREC(JTE2)} = -j\eta\hat{Y}_{ii}^{2,2\mathrm{TE},2\mathrm{TE}} \tag{3.48}$$

where both $-j\eta\hat{Y}_{ii}^{1,1\mathrm{TE},1\mathrm{TE}}$ and $-j\eta\hat{Y}_{ii}^{2,2\mathrm{TE},2\mathrm{TE}}$ are given by the right-hand side of eq. (2.12) of [2] with δ_{ij} deleted.

If P = 1 or Q = 1 so that, according to (3.30) and (3.31), $p = 0$ or $q = 0$, then there are no TM matrix elements. If $p \neq 0$ and $q \neq 0$, then the five statements after statement 17 are executed. The last two of these statements set

$$\mathrm{YREC(JTM)} = -j\eta\hat{Y}_{ii}^{1,1\mathrm{TM},1\mathrm{TM}} \tag{3.49}$$

$$\mathrm{YREC(JTM2)} = -j\eta\hat{Y}_{ii}^{2,2\mathrm{TM},2\mathrm{TM}} \tag{3.50}$$

where both $-j\eta\hat{Y}_{ii}^{1,1\mathrm{TM},1\mathrm{TM}}$ and $-j\eta\hat{Y}_{ii}^{2,2\mathrm{TM},2\mathrm{TM}}$ are given by the right-hand side of eq. (2.13) of [2] with δ_{ij} deleted. In (3.49),

$$\mathrm{JTM} = \mathrm{P} - 1 + \begin{cases} 0, & \mathrm{Q} = 2 \\ \displaystyle\sum_{l=2}^{\mathrm{Q}-1}(\mathrm{MM}(l) - 1), & \mathrm{Q} > 2. \end{cases} \tag{3.51}$$

The right-hand side of (3.51) is the subscript i that appears in $\hat{Y}_{ii}^{1,1\mathrm{TM},1\mathrm{TM}}$.

3.4 Quantities That Depend on the Modes of the Circular Waveguide

In this section, the admittance matrix of the circular waveguide, the excitation vector, and the normalized amplitudes of the propagating circular waveguide modes due each expansion function are calculated. The admittance matrix of the circular waveguide is Y^3 given by (1.38). The excitation vector is the column vector of the $\vec{I}$'s on the right-hand side of (1.32). The normalized amplitudes of the TM_{01}^{e} propagating circular waveguide mode due to the expansion functions $\underline{M}_{pq}^{\gamma\mathrm{TM}}$ and $\underline{M}_{pq}^{\gamma\mathrm{TE}}$ are, respectively, $C_{01,pq}^{\mathrm{TM}e,\gamma\mathrm{TM}}$ and $C_{01,pq}^{\mathrm{TM}e,\gamma\mathrm{TE}}$ given by eqs. (6.89) and (6.90) of [2]. The normalized amplitudes of the TE_{11}^{e} propagating circular waveguide mode due to the expansion functions $\underline{M}_{pq}^{\gamma\mathrm{TM}}$ and $\underline{M}_{pq}^{\gamma\mathrm{TE}}$ are, respectively, $C_{11,pq}^{\mathrm{TE}e,\gamma\mathrm{TM}}$ and $C_{11,pq}^{\mathrm{TE}e,\gamma\mathrm{TE}}$ given by eqs. (6.97) and (6.98) of [2].† The normalized amplitudes of the TE_{11}^{o} propagating circular waveguide mode due to the expansion functions $\underline{M}_{pq}^{\gamma\mathrm{TM}}$ and $\underline{M}_{pq}^{\gamma\mathrm{TE}}$ are, respectively, $C_{11,pq}^{\mathrm{TE}o,\gamma\mathrm{TM}}$ and $C_{11,pq}^{\mathrm{TE}o,\gamma\mathrm{TE}}$ given by eqs. (6.100) and (6.101) of [2]. Here, γ may be either 1 or 2. The superscript "e" attached to TM indicates that the z-directed electric field of the mode is even in ϕ, the superscript "e" attached to TE indicates that the z-directed magnetic field of the mode is even in ϕ, and the superscript "o" attached to TE indicates that the z-directed magnetic field of the mode is odd in ϕ. The elements of the admittance matrix depend on quantities associated with all circular waveguide modes. However, the elements of the excitation vector and the coefficients $C_{01,pq}^{\mathrm{TM}e,\gamma\mathrm{TM}}$ and $C_{01,pq}^{\mathrm{TM}e,\gamma\mathrm{TE}}$ depend on only TM_{01} quantities. The coefficients $C_{11,pq}^{\mathrm{TE}e,\gamma\mathrm{TM}}$, $C_{11,pq}^{\mathrm{TE}e,\gamma\mathrm{TE}}$, $C_{11,pq}^{\mathrm{TE}o,\gamma\mathrm{TM}}$, and $C_{11,pq}^{\mathrm{TE}o,\gamma\mathrm{TE}}$ depend on only TE_{11} quantities.

On the right-hand side of (1.38), the ij^{th} elements of the Y's are given implicitly‡ by eqs. (3.1)–(3.4) of [2]. Formulas for the quantities on the right-hand sides of eqs. (3.1)–(3.4) of [2] are given in Chapter 3 of [2]. The excitation vector and the normalized amplitudes of the TM_{01}^{e} circular waveguide mode due to the expansion functions will be calculated along with the terms for which $r = 0$ and $s = 1$ on the right-hand sides of eqs. (3.1)–(3.4) of [2]. The elements of the excitation vector are given by eqs. (4.8) and (4.9) of [2]. The normalized amplitudes of the TE_{11}^{e} and TE_{11}^{o} circular waveguide modes due to the expansion functions will be calculated along with the terms for which $r = s = 1$ on the right-hand sides of eqs. (3.1)–(3.4) of [2].

†There are misprints in eqs. (6.97), (6.98), (6.100), and (6.101) of [2]. The left-hand sides of these equations should be $C_{11,pq}^{\mathrm{TE}e,\gamma\mathrm{TM}}$, $C_{11,pq}^{\mathrm{TE}e,\gamma\mathrm{TE}}$, $C_{11,pq}^{\mathrm{TE}o,\gamma\mathrm{TM}}$, and $C_{11,pq}^{\mathrm{TE}o,\gamma\mathrm{TE}}$, respectively.

‡Equations (3.1)–(3.4) of [2] give the products of these elements with $-j\eta$ rather than these elements themselves.

3.4.1 Preliminary Calculations Independent of r, s, i, and j

The statement after statement 13 and the 14 statements before DO loop 12 define variables that do not depend on the summation indices r and s and the subscripts i and j in eqs. (3.1)–(3.4) of [2]. These statements set

$$\mathrm{K2} = 2{*}(\mathrm{KTM} + \mathrm{KTE}) \tag{3.52}$$
$$\mathrm{C5} = \frac{c}{2a} \tag{3.53}$$
$$\mathrm{ZSS} = \frac{2a}{c} \tag{3.54}$$
$$\mathrm{PMAX} = \mathrm{MM}(1) \tag{3.55}$$
$$\mathrm{XZ} = \frac{x_o}{a} \tag{3.56}$$
$$\mathrm{TZTM} = 8\phi_o\sqrt{\frac{b}{\pi c}} \tag{3.57}$$
$$\mathrm{TZTE} = 8\phi_o\sqrt{\frac{c}{\pi b}} \tag{3.58}$$
$$\mathrm{TA} = \frac{2\pi}{ka} \tag{3.59}$$
$$\mathrm{SQ2} = \frac{1}{\sqrt{2}} \tag{3.60}$$
$$\mathrm{TC1} = \sqrt{\frac{\pi b}{c}} \tag{3.61}$$
$$\mathrm{TC5} = \sqrt{\frac{2\pi b}{c}} \tag{3.62}$$
$$\mathrm{TC1} = ka\sqrt{\frac{\pi b}{c}} \tag{3.63}$$
$$\mathrm{SN1} = \sin\phi_o \tag{3.64}$$
$$\mathrm{CS1} = \cos\phi_o \tag{3.65}$$
$$\mathrm{K3} = 2(\mathrm{KTM} + \mathrm{KTE})^2. \tag{3.66}$$

3.4.2 Overview of the Calculation of the Circular Waveguide Admittance Matrix

The elements of the normalized admittance matrix $-j\eta Y^3$ will be stored by columns in the one-dimensional array Y. Each element of the first K1 columns of $-j\eta Y^3$ will be accumulated in its assigned location in Y. The last K1 columns of $-j\eta Y^3$ are given by

$$-j\eta \begin{bmatrix} Y^{3,\,1\mathrm{TM},2\mathrm{TM}} & Y^{3,1\mathrm{TM},2\mathrm{TE}} \\ Y^{3,\,1\mathrm{TE},2\mathrm{TM}} & Y^{3,1\mathrm{TE},2\mathrm{TE}} \\ Y^{3,\,2\mathrm{TM},2\mathrm{TM}} & Y^{3,2\mathrm{TM},2\mathrm{TE}} \\ Y^{3,\,2\mathrm{TE},2\mathrm{TM}} & Y^{3,2\mathrm{TE},2\mathrm{TE}} \end{bmatrix} = -j\eta \begin{bmatrix} Y^{3,\,2\mathrm{TM},1\mathrm{TM}} & Y^{3,2\mathrm{TM},1\mathrm{TE}} \\ Y^{3,\,2\mathrm{TE},1\mathrm{TM}} & Y^{3,2\mathrm{TE},1\mathrm{TE}} \\ Y^{3,\,1\mathrm{TM},1\mathrm{TM}} & Y^{3,1\mathrm{TM},1\mathrm{TE}} \\ Y^{3,\,1\mathrm{TE},1\mathrm{TM}} & Y^{3,1\mathrm{TE},1\mathrm{TE}} \end{bmatrix}. \quad (3.67)$$

To arrive at relationship (3.67), note that the only quantities on the right-hand sides of eqs. (3.1)–(3.4) of [2] that depend on α and γ are the $\hat{S}$'s. The only quantities on the right-hand sides of eqs. (3.10)–(3.13) of [2] for the $\hat{S}$'s that depend on α and γ are the $\hat{\phi}$'s given by eqs. (3.28)–(3.31) of [2]. Using eqs. (3.32)–(3.35) of [2], we see that the $\hat{\phi}$'s of eqs. (3.28)–(3.31) of [2] are the same at $\alpha = \gamma = 2$ as they are at $\alpha = \gamma = 1$. Because of eqs. (3.36)–(3.39) of [2], the $\hat{\phi}$'s of (3.28)–(3.31) of [2] are the same at $\alpha = 1$ and $\gamma = 2$ as they are at $\alpha = 2$ and $\gamma = 1$. Therefore, (3.67) holds.

Before the elements of the first K1 columns of $-j\eta Y^3$ are accumulated in their assigned locations in Y, DO loop 12 sets the contents of these locations equal to zero. The accumulations are done in nested DO loops 19, 20, 21, 22, 23, 24, and 28. These DO loops are arranged as follows.

```
      DO 19 R=1,500
C     CALCULATIONS INVOLVING R
      DO 20 S=1,SMAX
C     CALCULATIONS INVOLVING R AND S
      DO 21 N=1,NMAX
C     CALCULATIONS INVOLVING R, S, AND N
      DO 22 Q=1,NMAX
C     CALCULATIONS INVOLVING R, S, N, AND Q
      DO 23 M=M2,M3
C     CALCULATIONS INVOLVING R, S, N, Q, AND M
      DO 24 P=P2,P3
C     CALCULATIONS INVOLVING R, S, N, Q, M, AND P
      DO 28 J=1,2
```

```
C       CALCULATIONS INVOLVING R, S, N, Q, M, P, AND J
   28 CONTINUE
   24 CONTINUE
   23 CONTINUE
   22 CONTINUE
   21 CONTINUE
   20 CONTINUE
   19 CONTINUE
```

Here,

$$\mathrm{SMAX} = s_{\max} \tag{3.68}$$

where $s_{\max}$ appears in (2.14). Moreover, NMAX appears in (3.5). Furthermore,

$$\mathrm{M2} = \begin{cases} 2, & \mathrm{N} = 1 \\ 1, & \mathrm{N} > 1 \end{cases} \tag{3.69}$$

$$\mathrm{M3} = \mathrm{MM(N)} \tag{3.70}$$

$$\mathrm{P2} = \begin{cases} 2, & \mathrm{Q} = 1 \\ 1, & \mathrm{Q} > 1 \end{cases} \tag{3.71}$$

$$\mathrm{P3} = \mathrm{MM(Q)} \tag{3.72}$$

where MM apears in (3.5) with the argument $n+1$ rather than N or Q. The indices R, S, and J of DO loops 19, 20, and 28 are related to the summation indices r and s and the superscript α in eqs. (3.1)-(3.4) of [2] by

$$\mathrm{R} = r + 1 \tag{3.73}$$

$$\mathrm{S} = s \tag{3.74}$$

$$\mathrm{J} = \alpha. \tag{3.75}$$

The indices Q and P of DO loops 22 and 24 are related to the subscript j in eqs. (3.1) and (3.2) of [2] by

$$j = \mathrm{P} - 1 + \begin{cases} 0, & \mathrm{Q} = 2 \\ \sum_{l=2}^{\mathrm{Q}-1} (\mathrm{MM}(l) - 1), & \mathrm{Q} > 2 \end{cases} \tag{3.76}$$

for

$$\left.\begin{array}{l}\mathrm{Q} = 2, 3, \cdots, \mathrm{NMAX} \\ \mathrm{P} = 2, 3, \cdots, \mathrm{MM(Q)}\end{array}\right\}. \tag{3.77}$$

Alternatively, the indices Q and P of DO loops 22 and 24 are related to the subscript j in eqs. (3.3) and (3.4) of [2] by

$$j = \mathrm{P} - 1 + \begin{cases} 0, & \mathrm{Q} = 1 \\ \sum\limits_{l=1}^{\mathrm{Q}-1} \mathrm{MM}(l), & \mathrm{Q} > 1 \end{cases} \tag{3.78}$$

for

$$\left.\begin{array}{l}\mathrm{Q} = 1, 2, \cdots, \mathrm{NMAX} \\ \mathrm{P} = \mathrm{P2}, \mathrm{P2}+1, \cdots, \mathrm{MM(Q)}\end{array}\right\} \tag{3.79}$$

where P2 is given by (3.71). The right-hand sides of (3.76) and (3.78) are those of (3.51) and (3.32), respectively.

Equations (3.76) and (3.77) define Q and P in terms of j because if you know j, you can use (3.76) and (3.77) to determine Q and P uniquely. The indices Q and P of DO loops 22 and 24 are more simply defined by

$$\mathrm{Q} = q + 1 \tag{3.80}$$

$$\mathrm{P} = p + 1 \tag{3.81}$$

where q and p appear in Chapter 3 of [2]. All the combinations of Q and P that are obtained in nested DO loops 22 and 24 appear in (3.79). However, neither P = 1 nor Q = 1 appear in (3.77). The subscript j and the superscript γTM common to both $Y_{ij}^{3,\alpha\mathrm{TM},\gamma\mathrm{TM}}$ and $Y_{ij}^{3,\alpha\mathrm{TE},\gamma\mathrm{TM}}$ of eqs. (3.1) and (3.2) of [2] indicate the TM expansion functions $\underline{M}_{pq}^{\gamma\mathrm{TM}}(\phi, z)$ of (1.6) and (1.7). These expansion functions exist only for $p \geq 1$ and $q \geq 1$.

Whereas the indices Q and P of DO loops 22 and 24 were related to j in eqs. (3.1)–(3.4) of [2], the indices N and M of DO loops 21 and 23 are related to i. The indices N and M of DO loops 21 and 23 are related to the subscript i in eqs. (3.1) and (3.3) of [2] by

$$i = \mathrm{M} - 1 + \begin{cases} 0, & \mathrm{N} = 2 \\ \sum\limits_{l=2}^{\mathrm{N}-1} (\mathrm{MM}(l) - 1), & \mathrm{N} > 2 \end{cases} \tag{3.82}$$

for

$$\left.\begin{array}{l}\mathrm{N} = 2,3,\cdots,\mathrm{NMAX} \\ \mathrm{M} = 2,3,\cdots,\mathrm{MM(N)}\end{array}\right\}. \tag{3.83}$$

Alternatively, the indices N and M of DO loops 21 and 23 are related to the subscript i in eqs. (3.2) and (3.4) of [2] by

$$i = \mathrm{M} - 1 + \begin{cases} 0, & \mathrm{N} = 1 \\ \sum\limits_{l=1}^{\mathrm{N}-1} \mathrm{MM}(l), & \mathrm{N} > 1 \end{cases} \tag{3.84}$$

for

$$\left.\begin{array}{l}\mathrm{N} = 1,2,\cdots,\mathrm{NMAX} \\ \mathrm{M} = \mathrm{M2},\mathrm{M2}+1,\cdots,\mathrm{MM(N)}\end{array}\right\} \tag{3.85}$$

where M2 is given by (3.69). The indices N and M of DO loops 21 and 23 are related to n and m of Chapter 3 of [2] by

$$\mathrm{N} = n + 1 \tag{3.86}$$

$$\mathrm{M} = m + 1\,. \tag{3.87}$$

Neither M = 1 nor N = 1 appear in (3.83). The subscript i and the superscript αTM common to both $Y_{ij}^{3,\alpha\mathrm{TM},\gamma\mathrm{TM}}$ and $Y_{ij}^{3,\alpha\mathrm{TM},\gamma\mathrm{TE}}$ of eqs. (3.1) and (3.3) of [2] indicate that the matrix element is the result of taking the symmetric product of either (1.4) with the TM expansion function $\underline{M}_{mn}^{1\mathrm{TM}}$ or (1.5) with the TM expansion function $\underline{M}_{mn}^{2\mathrm{TM}}$. In either case, $m \geq 1$ and $n \geq 1$.

3.4.3 Calculations Involving r

In view of (3.73), calculations involving r are calculations involving R. As indicated in the arrangement of DO loops shown in Section 3.4.2, these calculations occur in DO loop 19 but not in DO loop 20. They are performed by the first seven statements in the range of DO loop 19. The first three statements in DO loop 19 set

$$\mathrm{SGR} = (-1)^r \tag{3.88}$$

$$\mathrm{R1} = r \tag{3.89}$$

$$\mathrm{RS} = r^2. \tag{3.90}$$

The statement

$$\text{CALL BES(R,XJ,XP)}$$

sets

$$\text{XJ}(s) = x_{rs} \text{ for } s = 1, 2, \cdots, s_{\max} \tag{3.91}$$

$$\text{XJP}(s) = x'_{rs} \text{ for } s = 1, 2, \cdots, s_{\max} \tag{3.92}$$

$$\text{SMAX} = s_{\max} \tag{3.93}$$

where x_{rs} is defined by (2.8), x'_{rs} is defined by (2.11), and $s_{\max}$ appears in (2.14).† The fifth statement in DO loop 19 terminates execution if $s_{\max} > 200$.

If $s_{\max} = 0$, the sixth statement in DO loop 19 sends execution to statement 25 beyond the range of DO loop 19. Now, $s_{\max} = 0$ only when $r = r_{\max} + 1$‡ so that the effect of the sixth statement in DO loop 19 is to send execution out of DO loop 19 when $r = r_{\max} + 1$ if, recalling (3.73) where the index R of DO loop 19 cannot exceed 500, $r_{\max} + 2 \leq 500$. If $r_{\max} + 2 > 500$, then, because of the "500" in the statement

DO 19 R=1,500,

normal termination of DO loop 19 will occur when $r = 499 < r_{\max} + 1$.

The statement

CALL PHI

puts $\phi_p^{(1)}$ through $\phi_p^{(4)}$ of eqs. (3.40)–(3.43) of [2] in PH1$(p+1)$, PH2$(p+1)$, PH3$(p+1)$, and PH4$(p+1)$, respectively, for $\{p = 0, 1, 2, \cdots, \text{PMAX} - 1\}$ where PMAX is given by (3.55). This statement also puts $\phi^{\alpha 1 \gamma 1}$, $\phi^{\alpha 2 \gamma 1}$, $\phi^{\alpha 1 \gamma 2}$, and $\phi^{\alpha 2 \gamma 2}$ of eqs. (3.36)–(3.39) of [2] in PH1$(m + 1 + \text{PMAX})$, PH3$(m + 1 + \text{PMAX})$, PH2$(m + 1 + \text{PMAX})$, and PH4$(m + 1 + \text{PMAX})$, respectively, for $\{m = 0, 1, 2, \cdots, \text{PMAX} - 1\}$.

3.4.4 Calculations Involving r and s

In view of (3.73) and (3.74), calculations involving r and s are calculations involving R and S. As indicated in the arrangement of DO loops shown in Section 3.4.2, these calculations occur in DO loop 20 but not in DO loop 21.

†To be exact, (3.91) and (3.92) are executed for the range of values of s in (6.4) where $S_{\max}$ might be $s_{\max} + 1$. However, XJ(s) and XJP(s) will be used only for $\{s = 1, 2, \cdots, s_{\max}\}$ in the main program.

‡Here, $r_{\max}$ first appears in (2.14) and is defined shortly thereafter.

They are performed by the first 144 statements in DO loop 20, all of those statements of DO loop 20 prior to the statement

DO 21 N=1,NMAX.

The first statement in DO loop 20 always sets[†]

$$\mathrm{XXTM} = x_{rs}^2. \tag{3.94}$$

If $x_{rs} < ka$, then the first statement in DO loop 20 sets XXTM of (3.94) and

$$\mathrm{ITM} = 1 \tag{3.95}$$

$$\mathrm{GAMTM} = \beta_{rs}^{TM} a \tag{3.96}$$

$$\mathrm{TMP(N)} = n^{\mathrm{TM}+} c \tag{3.97}$$

$$\mathrm{TMM(N)} = n^{\mathrm{TM}-} c \tag{3.98}$$

$$\mathrm{DTM(N)} = \hat{D}_n^{\mathrm{TM}} \tag{3.99}$$

$$\mathrm{GTM(N)} = \hat{G}_n^{\mathrm{TM}} \tag{3.100}$$

where $\beta_{rs}^{\mathrm{TM}} a$ is given by eq. (3.59) of [2]. Moreover, $n^{\mathrm{TM}+}c$, $n^{\mathrm{TM}-}c$, and $\hat{G}_n^{\mathrm{TM}}$ are given, respectively, by eqs. (3.79), (3.78), and (3.80) of [2] with δ and q replaced by TM and n, respectively. Furthermore, $\hat{D}_n^{\mathrm{TM}}$ is given by eq. (3.82) of [2] with δ replaced by TM. In (3.97)-(3.100),

$$\mathrm{N} = n + 1, \quad \text{and} \quad \mathrm{N} = 1, 2, \cdots, \mathrm{NMAX}. \tag{3.101}$$

If $x_{rs} \geq ka$, then the first statement in DO loop 20 sets XXTM of (3.94) and

$$\mathrm{ITM} = 2 \tag{3.102}$$

$$\mathrm{GAMTM} = \gamma_{rs}^{\mathrm{TM}} a \tag{3.103}$$

$$\mathrm{DQTM(N)} = \frac{1}{(n\pi)^2 + (\gamma_{rs}^{\mathrm{TM}} c)^2} \tag{3.104}$$

$$\mathrm{GCSTM} = \left(\gamma_{rs}^{\mathrm{TM}} c\right)^2 \tag{3.105}$$

$$\mathrm{GC2TM} = 2\gamma_{rs}^{\mathrm{TM}} c \tag{3.106}$$

$$\mathrm{ZEETM} = -4 z_{ee}^{\mathrm{TM}} \tag{3.107}$$

$$\mathrm{ZZTM} = -4 z_o^{\mathrm{TM}} \tag{3.108}$$

[†]See Section 9.2 where the output variables of the subroutine DGN are defined in terms of the input variables introduced in Section 9.1.

$$\text{ZOETM} = -4z_{oe}^{\text{TM}} \tag{3.109}$$

$$\text{ZOOTM} = -4z_{oo}^{\text{TM}} \tag{3.110}$$

$$\text{PGC} = \frac{\pi}{\gamma_{rs}^{\text{TM}} c} \tag{3.111}$$

where $\gamma_{rs}^{\text{TM}} a$ is given by eq. (3.57) of [2].† Furthermore, z_{ee}^{TM}, z_{o}^{TM}, z_{oe}^{TM}, and z_{oo}^{TM} are, respectively, z_{ee}, z_o, z_{oe}, and z_{oo} of eqs. (3.99)–(3.102) of [2] with g replaced by γ_{rs}^{TM}. In (3.104), N is related to n by (3.101).

The second statement in DO loop 20 always sets

$$\text{XXTE} = x_{rs}'^2 . \tag{3.112}$$

If $x'_{rs} < ka$, then the second statement in DO loop 20 sets XXTE of (3.112) and

$$\text{ITE} = 1 \tag{3.113}$$

$$\text{GAMTE} = \beta_{rs}^{TE} a \tag{3.114}$$

$$\text{TEP(N)} = n^{\text{TE}+} c \tag{3.115}$$

$$\text{TEM(N)} = n^{\text{TE}-} c \tag{3.116}$$

$$\text{DTE(N)} = \hat{D}_n^{\text{TE}} \tag{3.117}$$

$$\text{GTE(N)} = \hat{G}_n^{\text{TE}} \tag{3.118}$$

$$\text{D3(N)} = \hat{D}_n^{(3)} \tag{3.119}$$

$$\text{G4(N)} = \hat{G}_n^{(4)} \tag{3.120}$$

where $\beta_{rs}^{\text{TE}} a$ is given by eq. (3.60) of [2] and $\hat{D}_n^{(3)}$ is given by eq. (3.111) of [2]. Moreover, $n^{\text{TE}+}c$, $n^{\text{TE}-}c$, and $\hat{G}_n^{\text{TE}}$ are given, respectively, by eqs. (3.79), (3.78), and (3.80) of [2] with δ and q replaced by TE and n, respectively. Furthermore, $\hat{D}_n^{\text{TE}}$ is given by eq. (3.82) of [2] with δ replaced by TE. Finally, $\hat{G}_n^{(4)}$ is given by eq. (3.109) of [2] with q replaced by n. In (3.115)–(3.120), N is related to n by (3.101). If $x'_{rs} \geq ka$, then the second statement in DO loop 20 sets XXTE of (3.112) and

$$\text{ITE} = 2 \tag{3.121}$$

$$\text{GAMTE} = \gamma_{rs}^{TE} a \tag{3.122}$$

†Actually, x_{rs} should not be exactly equal to ka. If x_{rs} were exactly equal to ka, then γ_{rs}^{TM} would be zero in which case PGC could not be calculated.

$$\mathrm{DQTE(N)} = \frac{1}{(n\pi)^2 + \left(\gamma_{rs}^{\mathrm{TE}}c\right)^2} \tag{3.123}$$

$$\mathrm{GCSTE} = \left(\gamma_{rs}^{\mathrm{TE}}c\right)^2 \tag{3.124}$$

$$\mathrm{GC2TE} = 2\gamma_{rs}^{\mathrm{TE}}c \tag{3.125}$$

$$\mathrm{ZEETE} = -4z_{ee}^{\mathrm{TE}} \tag{3.126}$$

$$\mathrm{ZZTE} = -4z_{o}^{\mathrm{TE}} \tag{3.127}$$

$$\mathrm{ZOETE} = -4z_{oe}^{\mathrm{TE}} \tag{3.128}$$

$$\mathrm{ZOOTE} = -4z_{oo}^{\mathrm{TE}} \tag{3.129}$$

$$\mathrm{PGC} = \frac{\pi}{\gamma_{rs}^{\mathrm{TE}}c} \tag{3.130}$$

where $\gamma_{rs}^{\mathrm{TE}}a$ is given by eq. (3.58) of [2]. Furthermore, z_{ee}^{TE}, z_{o}^{TE}, z_{oe}^{TE}, and z_{oo}^{TE} are, respectively, z_{ee}, z_o, z_{oe}, and z_{oo} of eqs. (3.99)–(3.102) of [2] with g replaced by $\gamma_{rs}^{\mathrm{TE}}$. In (3.123), N is related to n by (3.101). The variable PGC of (3.130) supersedes PGC of (3.111); PGC of (3.111) is never used in the main program. However, PGC of (3.130) is used.†

Substituting eqs. (3.66)–(3.70) of [2] into eqs. (3.52)–(3.56) of [2] and using (3.57), (3.58), and eq. (F.121) of [1], which is

$$z_{ss}^{\mathrm{TE}} = \begin{cases} \dfrac{c}{2}, & n = q \neq 0 \\ 0, & \text{otherwise,} \end{cases} \tag{3.131}$$

we obtain

$$z_1 = (\mathrm{W1})\left\{\begin{array}{ll} 1, & x_{rs} < ka \\ j, & x_{rs} \geq ka \end{array}\right\}(F^{\mathrm{TM}} + \hat{G}_q^{\mathrm{TM}}\hat{D}_n^{\mathrm{TM}}) \tag{3.132}$$

$$z_2 = (\mathrm{W2})\left\{\begin{array}{ll} 1, & x'_{rs} < ka \\ -j, & x'_{rs} \geq ka \end{array}\right\}(F^{\mathrm{TE}} + \hat{G}_q^{\mathrm{TE}}\hat{D}_n^{\mathrm{TE}}) \tag{3.133}$$

$$z_3 = (\mathrm{W3})(F^{(3)} + \hat{G}_q^{\mathrm{TE}}\hat{D}_n^{(3)}) \tag{3.134}$$

$$z_4 = -(\mathrm{W3})(F^{(4)} + \hat{G}_q^{(4)}\hat{D}_n^{\mathrm{TE}}) \tag{3.135}$$

$$z_5 = (\mathrm{W6})\left\{\begin{array}{ll} 1, & n = q \neq 0 \\ 0, & \text{otherwise} \end{array}\right\}$$

†See the statement before statement 86.

$$+(\mathrm{W5})\left\{\begin{array}{ll} 1, & x'_{rs} < ka \\ j, & x'_{rs} \geq ka \end{array}\right\}(F^{(5)} + \hat{G}_q^{(4)}\hat{D}_n^{(3)}) \tag{3.136}$$

where

$$\mathrm{W1} = \left(\frac{\epsilon_r}{2}\right)(ka)^2\left\{\begin{array}{ll} \dfrac{1}{\beta_{rs}^{\mathrm{TM}}a}, & x_{rs} < ka \\ \dfrac{1}{\gamma_{rs}^{\mathrm{TM}}a}, & x_{rs} \geq ka \end{array}\right\} \tag{3.137}$$

$$\mathrm{W2} = -\frac{r^2}{x'^2_{rs} - r^2}\left\{\begin{array}{ll} \beta_{rs}^{\mathrm{TE}}a, & x'_{rs} < ka \\ \gamma_{rs}^{\mathrm{TE}}a, & x'_{rs} \geq ka \end{array}\right\} \tag{3.138}$$

$$\mathrm{W3} = \left(\frac{r x'^2_{rs}}{x'^2_{rs} - r^2}\right)\left(\frac{x_o}{a}\right) \tag{3.139}$$

$$\mathrm{W5} = \left(\frac{\epsilon_r}{2}\right)\left(\frac{x'^4_{rs}}{x'^2_{rs} - r^2}\right)\left(\frac{x_o}{a}\right)^2\left\{\begin{array}{ll} \dfrac{1}{\beta_{rs}^{\mathrm{TE}}a}, & x'_{rs} < ka \\ \dfrac{1}{\gamma_{rs}^{\mathrm{TE}}a}, & x'_{rs} \geq ka \end{array}\right\} \tag{3.140}$$

$$\mathrm{W6} = \left(\frac{\epsilon_r}{2}\right)\left(\frac{x'^4_{rs}}{x'^2_{rs} - r^2}\right)\left(\frac{x_o}{a}\right)^2\left(\frac{2a}{c x'^2_{rs}}\right). \tag{3.141}$$

Here, ϵ_r is Neumann's number given by

$$\epsilon_r = \left\{\begin{array}{ll} 1, & r = 0 \\ 2, & r = 1, 2, \cdots. \end{array}\right. \tag{3.142}$$

In obtaining (3.135), we omitted the factor $\epsilon_r/2$ in eq. (3.55) of [2]. This factor is superfluous because it is multiplied by r. In (3.137)–(3.141), x_o/a, x_{rs}, and x'_{rs} are, according to eqs. (2.8), (3.49), and (3.50) of [2], given by

$$\frac{x_o}{a} = \frac{\sin\phi_o}{\phi_o} \tag{3.143}$$

$$x_{rs} = k_{rs}^{\mathrm{TM}}a \tag{3.144}$$

$$x'_{rs} = k_{rs}^{\mathrm{TE}}a. \tag{3.145}$$

The thirteen statements before statement 46 in DO loop 20 set W1, W2, W3, W5, and W6 equal to the right-hand sides of (3.137)–(3.141), respectively.

If $r = 0$ and $s = 1$, control statement 46 and the control statement following it send execution to the next statement and eventually to DO loops 29 and 77. However, if $r = s = 1$, then the two previously mentioned control statements send execution to statement 71. For all other values† of r and s, execution passes from statement 46 directly to statement 68.

DO loop 29. which is executed only when $r = 0$ and s=1, calculates the elements of the excitation vector. The excitation vector is the column vector on the right-hand side of (1.32). The i^{th} elements of $\vec{I}^{1\text{TM}}$, $\vec{I}^{1\text{TE}}$, $\vec{I}^{2\text{TM}}$, and $\vec{I}^{2\text{TE}}$ therein are $I_i^{1\text{TM}}$, $I_i^{1\text{TE}}$, $I_i^{2\text{TM}}$, and $I_i^{2\text{TE}}$ given by eqs. (4.8) and (4.9) of [2]. In nested DO loops 29 and 52, whose indices N and M take on the same values as the indices N and M of nested DO loops 21 and 23,‡ TI(MTM) and TI(MTM + K1) are set equal to the right-hand side of eq. (4.8) of [2] provided that neither N nor M is 1. Furthermore, TI(MTE + KTM) and TI(MTE + KTM + K1) are set equal to the right-hand side of eq. (4.9) of [2] for all values of N and M in DO loops 29 and 52. Here, MTM and MTE are given by the right-hand sides of (3.82) and (3.84):

$$\text{MTM} = \text{M} - 1 + \begin{cases} 0, & N = 2 \\ \sum_{l=2}^{N-1} (\text{MM}(l) - 1), & N > 2 \end{cases} \tag{3.146}$$

and

$$\text{MTE} = \text{M} - 1 + \begin{cases} 0, & N = 1 \\ \sum_{l=1}^{N-1} \text{MM}(l), & N > 1\,. \end{cases} \tag{3.147}$$

Moreover, KTM and K1 are given by (4.14) and (3.10), respectively. In (3.10), KTE is given by (4.13).

The indices N and M of DO loops 29 and 52 are related to n and m in eqs. (4.8) and (4.9) of [2] by

$$\text{N} = n + 1 \tag{3.148}$$

$$\text{M} = m + 1. \tag{3.149}$$

† "All other values" are all values other that those mentioned in the previous two sentences.

‡ See the arrangement of DO loops in Section 3.4.2 and eqs. (3.69) and (3.70).

The fourth, fifth, and eighth statements in DO loop 29 set

$$\mathrm{TITM} = 8\phi_o n\sqrt{\frac{b}{\pi c}} \tag{3.150}$$

$$\mathrm{TITE} = 8\phi_o\sqrt{\frac{\epsilon_n c}{2\pi b}}\,. \tag{3.151}$$

If m is even and neither n nor m is zero, execution passes to the fourth statement in DO loop 52. The fifth and seventh statements in DO loop 52 set

$$\mathrm{TI(MTM)} = 0 \tag{3.152}$$

$$\mathrm{TI(MTM + K1)} = 0 \tag{3.153}$$

when m is even and neither n nor m is zero. The second and fourth statements after statement 66 set

$$\mathrm{TI(MTE + KTM)} = 0 \tag{3.154}$$

$$\mathrm{TI(MTE + KTM + K1)} = 0 \tag{3.155}$$

when m is even.

If m is odd, the second statement in DO loop 52 sends execution to statement 65. The statement after statement 65 sets

$$\mathrm{F1} = \frac{\hat{G}_n^{\mathrm{TM}}}{k_{mn}b}\,. \tag{3.156}$$

If neither n nor m is zero, the sixth and eighth statements after statement 65 set

$$\mathrm{TI(MTM)} = \mathrm{SA} \tag{3.157}$$

$$\mathrm{TI(MTM + K1)} = \mathrm{SA} \tag{3.158}$$

when m is odd where, thanks to the fifth statement after statement 65, SA is the right-hand side of eq. (4.8) of [2] when m is odd. The second and fourth statements after statement 67 set

$$\mathrm{TI(MTE + KTM)} = \mathrm{SA} \tag{3.159}$$

$$\mathrm{TI(MTE + KTM + K1)} = \mathrm{SA} \tag{3.160}$$

where, by virtue of statement 67, SA is the right-hand side of eq. (4.9) of [2] when m is odd.

Nested DO loops 77 and 78 calculate the normalized amplitudes $C_{01,pq}^{\mathrm{TMe},\gamma\mathrm{TM}}$ and $C_{01,pq}^{\mathrm{TMe},\gamma\mathrm{TE}}$ of the TM_{01}^{e} circular waveguide mode due to the expansion functions $\underline{M}_{pq}^{\gamma\mathrm{TM}}$ and $\underline{M}_{pq}^{\gamma\mathrm{TE}}$, respectively. These amplitudes are given by eqs. (6.89) and (6.90) of [2]. The third statement before DO loop 77 sets

$$\mathrm{TC2} = \sqrt{\frac{\pi b}{c}}\left(\frac{k}{\beta_{01}^{\mathrm{TM}}}\right). \tag{3.161}$$

The variables N, N1, M, M1, M2, M3, MTE, and MTM in nested DO loops 77 and 78 take on the same values as in nested DO loops 29 and 52. The indices N and M of nested DO loops 77 and 78 are related to q and p of eqs. (6.89) and (6.90) of [2] by

$$\mathrm{N} = q+1 \tag{3.162}$$

$$\mathrm{M} = p+1. \tag{3.163}$$

Upon entry into DO loop 78,

$$\mathrm{TC3} = \sqrt{\frac{\epsilon_q \pi b}{2c}}\left(\frac{k}{\beta_{01}^{\mathrm{TM}}}\right)\left[\hat{G}_q^{\mathrm{TM}}\right]_{01} \tag{3.164}$$

where $\left[\hat{G}_q^{\mathrm{TM}}\right]_{01}$ is $\hat{G}_q^{\mathrm{TM}}$ when $r=0$ and $s=1$.

After execution of the fourth statement in DO loop 78,

$$\mathrm{TC4} = \sqrt{\frac{\pi b}{c}}\left(\frac{k}{\beta_{01}^{\mathrm{TM}}}\right)\epsilon_{pq}\left[\hat{G}_q^{\mathrm{TM}}\right]_{01}\phi_p^{(2)} \tag{3.165}$$

where, as in eq. (6.60) of [2],

$$\epsilon_{pq} = \frac{1}{k_{pq} b}\sqrt{\frac{\epsilon_p \epsilon_q}{4}}\,. \tag{3.166}$$

The eighth and tenth statements in DO loop 78 set

$$\mathrm{CVTME(MTM)} = C_{01,pq}^{\mathrm{TMe},1\mathrm{TM}} \tag{3.167}$$

$$\mathrm{CVTME(MTM+K1)} = C_{01,pq}^{\mathrm{TMe},2\mathrm{TM}} \tag{3.168}$$

where $C_{01,pq}^{\text{TMe,1TM}}$ and $C_{01,pq}^{\text{TMe,2TM}}$ are given by eq. (6.89) of [2] with γ replaced by 1 and 2, respectively.† The second and fourth second and fourth statements after statement 79 set

$$\text{CVTME(MTE + KTM)} = C_{01,pq}^{\text{TMe,1TE}} \tag{3.169}$$

$$\text{CVTME(MTE + KTM + K1)} = C_{01,pq}^{\text{TMe,2TE}} \tag{3.170}$$

where $C_{01,pq}^{\text{TMe,1TE}}$ and $C_{01,pq}^{\text{TMe,2TE}}$ are given by eq. (6.90) of [2] with γ replaced by 1 and 2, respectively. The statement after statement 77 sets

$$\text{GAM01} = \beta_{01}^{\text{TM}} a \tag{3.171}$$

for use in statement 71.

Statement 71 is executed only when R = 2 and S = 1, that is, when $r = s = 1$. From statement 71, execution eventually passes to nested DO loops 80 and 73. These nested DO loops calculate the normalized amplitudes $C_{11,pq}^{\text{TEe},\gamma\text{TM}}$, $C_{11,pq}^{\text{TEe},\gamma\text{TE}}$, $C_{11,pq}^{\text{TEo},\gamma\text{TM}}$, and $C_{11,pq}^{\text{TEo},\gamma\text{TE}}$ of the TE_{11}^{e} and TE_{11}^{o} circular waveguide modes due to the expansion functions. Since the third statement in DO loop 20 has already set

$$\text{XR} = x_{rs}'^2 - r^2, \tag{3.172}$$

statement 71 and the statement following it set

$$\text{TC2} = \sqrt{\frac{2\pi b}{c}}\sqrt{\frac{\beta_{11}^{\text{TE}}}{\beta_{01}^{\text{TM}}\,(x_{11}'^2 - 1)}} \tag{3.173}$$

$$\text{TC6} = \left(\frac{x_{11}'^2}{\beta_{11}^{\text{TE}} a}\right)\left(\frac{x_o}{a}\right). \tag{3.174}$$

Because of (3.143), the factor x_o/a in (3.174) is equal to the factor $(\sin\phi_o)/\phi_o$ in eqs. (6.97), (6.98), (6.100), and (6.101) of [2].

The variables N, N1, M, M1, M2, M3, MTE, and MTM in nested DO loops 80 and 73 take on the same values as in nested DO loops 29 and 52. The indices N and M of nested DO loops 80 and 73 are related to q and p of eqs. (6.97), (6.98), (6.100), and (6.101) of [2] by

$$\text{N} = q + 1 \tag{3.175}$$

$$\text{M} = p + 1. \tag{3.176}$$

†The right-hand sides of eqs. (6.89) and (6.90) of [2] do not depend on γ.

Upon entry into DO loop 73,

$$\mathrm{TC3} = \sqrt{\frac{\epsilon_q \pi b}{c}} \sqrt{\frac{\beta_{11}^{\mathrm{TE}}}{\beta_{01}^{\mathrm{TM}} \left(x_{11}'^2 - 1\right)}} \left[\hat{G}_q^{\mathrm{TE}}\right]_{11} \tag{3.177}$$

$$\mathrm{TC7} = \sqrt{\frac{\epsilon_q \pi b}{c}} \sqrt{\frac{\beta_{11}^{\mathrm{TE}}}{\beta_{01}^{\mathrm{TM}} \left(x_{11}'^2 - 1\right)}} \left(\frac{x_o}{a}\right) \left(\frac{x_{11}'^2}{\beta_{11}^{\mathrm{TE}} a}\right) \left[\hat{G}_q^{(4)}\right]_{11} . \tag{3.178}$$

Upon execution of statement 74,

$$\mathrm{TC4} = \sqrt{\frac{2\pi b}{c}} \sqrt{\frac{\beta_{11}^{\mathrm{TE}}}{\beta_{01}^{\mathrm{TM}} \left(x_{11}'^2 - 1\right)}} \epsilon_{pq} \left[\hat{G}_q^{\mathrm{TE}}\right]_{11} \tag{3.179}$$

$$\mathrm{TC8} = \sqrt{\frac{2\pi b}{c}} \sqrt{\frac{\beta_{11}^{\mathrm{TE}}}{\beta_{01}^{\mathrm{TM}} \left(x_{11}'^2 - 1\right)}} \epsilon_{pq} \left(\frac{x_o}{a}\right) \left(\frac{x_{11}'^2}{\beta_{11}^{\mathrm{TE}} a}\right) \left[\hat{G}_q^{(4)}\right]_{11} . \tag{3.180}$$

The fourth, fifth, sixth, and seventh statements after statement 74 set

$$\mathrm{PAG1} = (\mathrm{TC4})\phi_p^{b1} \tag{3.181}$$

$$\mathrm{PAG2} = (\mathrm{TC4})\phi_p^{b2} \tag{3.182}$$

$$\mathrm{PAG3} = (\mathrm{TC8})\phi_p^{b3} \tag{3.183}$$

$$\mathrm{PAG4} = (\mathrm{TC8})\phi_p^{b4} \tag{3.184}$$

where TC4 and TC8 are given by (3.179) and (3.180). Moreover, ϕ_p^{b1}, ϕ_p^{b2}, ϕ_p^{b3}, and ϕ_p^{b4} are given by eqs. (6.105)–(6.108) of [2].†

The fifth and fourth statements before statement 72 set

$$\mathrm{CVTEE(MTM)} = C_{11,pq}^{\mathrm{TE}e,1\mathrm{TM}} \tag{3.185}$$

$$\mathrm{CVTEO(MTM)} = C_{11,pq}^{\mathrm{TE}o,1\mathrm{TM}} \tag{3.186}$$

where $C_{11,pq}^{\mathrm{TE}e,1\mathrm{TM}}$ and $C_{11,pq}^{\mathrm{TE}o,1\mathrm{TM}}$ are given, respectively, by eqs. (6.97) and (6.100) of [2] with $\gamma = 1$.‡ The second and first statements before statement

†There are misprints in eqs. (6.105)–(6.108) of [2]. Each subscript "o" on the right-hand sides of eqs. (6.105)–(6.108) of [2] should be replaced by "p". Furthermore, the first $\phi_p^{(3)}$ on the right-hand side of eq. (6.108) of [2] should be replaced by $\phi_p^{(4)}$.

‡As stated in a footnote of Section 3.4, there are misprints in eqs. (6.97), (6.98), (6.100), and (6.101) of [2]. The subscript "11" should be replaced by "$11, pq$" everywhere. Furthermore, the superscript "TEe" in eq. (6.101) of [2] should be replaced by "TEo".

72 set

$$\text{CVTEE(MTM + K1)} = C_{11,pq}^{\text{TE}e,2\text{TM}} \tag{3.187}$$

$$\text{CVTEO(MTM + K1)} = C_{11,pq}^{\text{TE}o,2\text{TM}} \tag{3.188}$$

where $C_{11,pq}^{\text{TE}e,2\text{TM}}$ and $C_{11,pq}^{\text{TE}o,2\text{TM}}$ are given, respectively, by eqs. (6.97) and (6.100) of [2] with $\gamma = 2$. The third and fourth statements after statement 72 set

$$\text{CVTEE(MTE + KTM)} = C_{11,pq}^{\text{TE}e,1\text{TE}} \tag{3.189}$$

$$\text{CVTEO(MTE + KTM)} = C_{11,pq}^{\text{TE}o,1\text{TE}} \tag{3.190}$$

where $C_{11,pq}^{\text{TE}e,1\text{TE}}$ and $C_{11,pq}^{\text{TE}o,1\text{TE}}$ are given, respectively, by eqs. (6.98) and (6.101) of [2] with $\gamma = 1$. The sixth and seventh statements after statement 72 set

$$\text{CVTEE(MTE + KTM + K1)} = C_{11,pq}^{\text{TE}e,2\text{TE}} \tag{3.191}$$

$$\text{CVTEO(MTE + KTM + K1)} = C_{11,pq}^{\text{TE}o,2\text{TE}} \tag{3.192}$$

where $C_{11,pq}^{\text{TE}e,2\text{TE}}$ and $C_{11,pq}^{\text{TE}o,2\text{TE}}$ are given, respectively, by the right-hand sides of eqs. (6.98) and (6.101) of [2] with $\gamma = 2$.

3.4.5 Calculations Involving r, s, and n

In view of (3.73), (3.74), and (3.86), calculations involving r, s, and n are calculations involving R, S, and N. As indicated in the arrangement of DO loops shown in Section 3.4.2, these calculations occur in DO loop 21 but not in DO loop 22. They are performed by the first 27 statements in DO loop 21, all of those statements of DO loop 21 prior to statement 86.

The meaning of the variables NTM and NTE, which are set to zero before entry into DO loop 21, is clarified by focusing on the following statements extracted from the main program:

```
68 NTM=0
   NTE=0
   DO 21 N=1,NMAX
   M2=1
```

```
      N1=N-1
      IF(N1.EQ.0) M2=2
      M3=MM(N)
   86 DO 22 Q=1,NMAX
   87 MTM=NTM
      MTE=NTE
      DO 23 M=M2,M3
      M1=M-1
      IF(N1.EQ.0.OR.M1.EQ.0) GO TO 26
      MTM=MTM+1
   26 MTE=MTE+1
   23 CONTINUE
   22 CONTINUE
      NTM=MTM
      NTE=MTE
   21 CONTINUE
```

Nested DO loops 21 and 23 were constructed so that the variables MTM and MTE therein take on the same values as those in nested DO loops 29 and 52. The variables NTM and NTE assure that, upon entry into DO loop 23, the values of MTM and MTE will be the same for all values of the index Q of DO loop 22. Here, DO loop 22 is regarded as merely an intervening DO loop, which causes DO loop 23 to be executed NMAX times for each value of N. After incremented in DO loop 23, MTM and MTE are equal, respectively, to i of (3.82) and i of (3.84).

The meaning of the variables QTM and QTE, which the fifth and sixth statements in DO loop 21 set to zero, is clarified by focusing on the following statements extracted from the main program:

```
      QTM=0
      QTE=0
   86 DO 22 Q=1, NMAX
      P2=1
      Q1=Q-1
      IF(Q1.EQ.0) P2=2
      P3=MM(Q)
      DO 23 M=M2,M3
      PTM=QTM
```

```
      PTE=QTE
      DO 24 P=P2,P3
      P1=P-1
      IF(Q1.EQ.0.OR.P1.EQ.0) GO TO 27
      PTM=PTM+1
   27 PTE=PTF+1
   24 CONTINUE
   23 CONTINUE
      QTM=PTM
      QTE=PTE
   22 CONTINUE
```

Here, DO loop 23 is regarded as merely an intervening DO loop. The limits M2 and M3 of its index are of no concern here. Note that the variables QTM, QTE, Q, Q1, P, P1, P2, P3, PTM, and PTE in the above FORTRAN statements take on the same values as the respective variables NTM, NTE, N, N1, M, M1, M2, M3, MTM, and MTE in the FORTRAN statements in the second paragraph of this section. After incremented in DO loop 24, PTM and PTE are equal, respectively, to j of (3.76) and j of (3.78).

The seventh and eighth statements in DO loop 21 set

$$\mathrm{FN1B} = \frac{nb}{c} \tag{3.193}$$

$$\mathrm{NEO} = \begin{cases} 1, & n \text{ even} \\ 2, & n \text{ odd.} \end{cases} \tag{3.194}$$

Statement 18 and the six statements following it set

$$\mathrm{TMMN} = n^{\mathrm{TM}-}c \tag{3.195}$$

$$\mathrm{TMPN} = n^{\mathrm{TM}+}c \tag{3.196}$$

$$\mathrm{DTMN} = \hat{D}_n^{\mathrm{TM}} \tag{3.197}$$

when $x_{rs} < ka$ and

$$\mathrm{DQTMN} = \frac{1}{(n\pi)^2 + (\gamma_{rs}^{\mathrm{TM}}c)^2} \tag{3.198}$$

$$\mathrm{TM1} = \frac{\left(\gamma_{rs}^{\mathrm{TM}}c\right)^2}{(n\pi)^2 + (\gamma_{rs}^{\mathrm{TM}}c)^2} \tag{3.199}$$

$$\text{TM2} = \frac{2\gamma_{rs}^{\text{TM}}c}{(n\pi)^2 + (\gamma_{rs}^{\text{TM}}c)^2} \tag{3.200}$$

when $x_{rs} \geq ka$. Statement 84 and the nine statements following it set

$$\text{TEMN} = n^{\text{TE}-}c \tag{3.201}$$
$$\text{TEPN} = n^{\text{TE}+}c \tag{3.202}$$
$$\text{DTEN} = \hat{D}_n^{\text{TE}} \tag{3.203}$$
$$\text{D3N} = \hat{D}_n^{(3)} \tag{3.204}$$

if $x'_{rs} < ka$ and

$$\text{DQTEN} = \frac{1}{(n\pi)^2 + (\gamma_{rs}^{\text{TE}}c)^2} \tag{3.205}$$

$$\text{TE1} = \frac{\left(\gamma_{rs}^{\text{TE}}c\right)^2}{(n\pi)^2 + (\gamma_{rs}^{\text{TE}}c)^2} \tag{3.206}$$

$$\text{TE2} = \frac{2\gamma_{rs}^{\text{TE}}c}{(n\pi)^2 + (\gamma_{rs}^{\text{TE}}c)^2} \tag{3.207}$$

$$\text{FN1} = n \tag{3.208}$$

$$\text{PNG} = \frac{n\pi}{\gamma_{rs}^{\text{TE}}c} \tag{3.209}$$

if $x'_{rs} \geq ka$.

3.4.6 Calculations Involving r, s, n, and q

In view of (3.73), (3.74), (3.86), and (3.80), calculations involving r, s, n, and q are calculations involving R, S, N, and Q. As indicated in the arrangement of DO loops shown in Section 3.4.2, these calculations occur in DO loop 22 but not in DO loop 23. They are performed by all statements in DO loop 22 prior to entry into DO loop 23.

The fifth through tenth statements in DO loop 22 set

$$\mathrm{W8} = W_8 \tag{3.210}$$

$$\mathrm{NQEO} = \begin{cases} 1, & n \text{ even}, \quad q \text{ even} \\ 2, & n \text{ even}, \quad q \text{ odd} \\ 2, & n \text{ odd}, \quad q \text{ even} \\ 3, & n \text{ odd}, \quad q \text{ odd} \end{cases} \tag{3.211}$$

$$\mathrm{NEQQ} = \begin{cases} 1, & n = q \neq 0 \\ 2, & \text{otherwise} \end{cases} \tag{3.212}$$

$$\mathrm{QPN} = \begin{cases} 1, & n = q = 0 \\ 2, & \text{otherwise.} \end{cases} \tag{3.213}$$

In (3.210), W_8 is given by eq. (3.6) of [2]. If $x_{rs} < ka$, statement 31 and the five statements following it set

$$\mathrm{IM} = n - q \tag{3.214}$$

$$\mathrm{IP} = n + q \tag{3.215}$$

$$\mathrm{TMMQ} = q^{\mathrm{TM}-}c \tag{3.216}$$

$$\mathrm{TMPQ} = q^{\mathrm{TM}+}c \tag{3.217}$$

$$\mathrm{FTM} = F^{\mathrm{TM}} \tag{3.218}$$

$$\mathrm{Z1} = z_1 \tag{3.219}$$

where F^{TM} is given by eq. (3.83) of [2] with δ replaced by TM. Moreover, z_1 is given by (3.132) for $x_{rs} < ka$. The function subroutine FXY called in the fourth statement after statement 31 is explained in Chapter 10.

If $x_{rs} \geq ka$, the block of statements beginning with statement 32 and ending with statement 120 sets

$$\mathrm{Z1R} = -c_{nq}^{\mathrm{TM}} \tag{3.220}$$

where c_{nq}^{TM} is given by the right-hand side of eq. (3.94) of [2] with δ replaced by TM in the definitions of the quantities therein. After execution of statement 122,

$$\mathrm{Z1R} = z_1 \tag{3.221}$$

where z_1 is given by (3.132) for $x_{rs} \geq ka$ with $j(F^{\mathrm{TM}} + \hat{G}_q^{\mathrm{TM}} \hat{D}_n^{\mathrm{TM}})$ given by eq. (3.93) of [2] with δ replaced by TM.

If $x'_{rs} < ka$, the block of statements beginning with statement 36 and ending with the second statement before statement 37 sets

$$\mathrm{Z2} = z_2 \tag{3.222}$$

$$\mathrm{Z3} = z_3 \tag{3.223}$$

$$\mathrm{Z4} = z_4 \tag{3.224}$$

$$\mathrm{Z5} = z_5 \tag{3.225}$$

where z_2, z_3, z_4, and z_5 are given for $x'_{rs} < ka$ by (3.133)–(3.136), respectively. In (3.133)–(3.136), F^{TE}, $F^{(3)}$, $F^{(4)}$, and $F^{(5)}$ are given, respectively, by eq. (3.83) of [2] with δ replaced by TE and eqs. (3.112)–(3.114) of [2].

If $x'_{rs} \geq ka$, the block of statements beginning with statement 37 and ending with statement 133 is executed. The block of statements beginning with statement 37 and ending with statement 128 sets

$$\mathrm{ZR} = -c_{nq}^{\mathrm{TE}} \tag{3.226}$$

where c_{nq}^{TE} is given by the right-hand side of eq. (3.94) of [2] with δ replaced by TE in the definitions of the quantities therein. The block of statements beginning with the statement after statement 128 and ending with statement 133 sets

$$\mathrm{Z2R} = z_2 \tag{3.227}$$

$$\mathrm{Z3R} = z_3 \tag{3.228}$$

$$\mathrm{Z4R} = z_4 \tag{3.229}$$

$$\mathrm{Z5R} = z_5 \tag{3.230}$$

where z_2 is given by (3.133) for $x'_{rs} \geq ka$ with $j(F^{\mathrm{TE}} + \hat{G}_q^{\mathrm{TE}} \hat{D}_n^{\mathrm{TE}})$ given by eq. (3.93) of [2] with δ replaced by TE. Furthermore, z_3, z_4, and z_5 are given, respectively, by (3.134), (3.135), and (3.136) for $x'_{rs} \geq ka$ with $F^{(3)} + \hat{G}_q^{\mathrm{TE}} \hat{D}_n^{(3)}$, $F^{(4)} + \hat{G}_q^{(4)} \hat{D}_n^{\mathrm{TE}}$ and $j(F^{(5)} + \hat{G}_q^{(4)} \hat{D}_n^{(3)})$ given, respectively, by eqs. (3.121), (3.128), and (3.132) of [2].

The variables Z1, Z2, Z3, Z4, and Z5 are complex variables whereas Z1R, Z2R, Z3R, Z4R, and Z5R are real variables. To avoid mixed modes later on, statement 53 and the three statements following it change the names of the real variables Z2R, Z3R, Z4R, and Z5R to Z2, Z3, Z4, and Z5 when $x_{rs} < ka$ and $x'_{rs} \geq ka$. Furthermore, statement 54 changes the name of Z1R to Z1 when $x_{rs} \geq ka$ and $x'_{rs} < ka$. The two statements before DO loop 23 appear in the first group of extracted FORTRAN statements in Section 3.4.5.

3.4.7 Calculations Involving r, s, n, q and m

In view of (3.73), (3.74), (3.86), (3.80), and (3.87), calculations involving r, s, n, q, and m are calculations involving R, S, N, Q, and M. As indicated in the arrangement of DO loops shown in Section 3.4.2, these calculations occur in DO loop 23 but not in DO loop 24. They are performed by all statements in DO loop 23 prior to entry into DO loop 24.

Before statement 26 is executed,

$$\mathrm{KMN} = \begin{cases} 1, & n = 0 \text{ or } m = 0 \\ 2, & \text{otherwise.} \end{cases} \tag{3.231}$$

Given m and n, KMN indicates whether $\underline{M}_{mn}^{1\mathrm{TM}}$ and $\underline{M}_{mn}^{2\mathrm{TM}}$ of (1.6) and (1.7) exist.† Since the subscript i in eq. (3.3) of [2] is a condensation of the subscripts m and n, KMN indicates whether $Y_{ij}^{3,1\mathrm{TM},1\mathrm{TE}}$ and $Y_{ij}^{3,2\mathrm{TM},1\mathrm{TE}}$ exist, given the values of m and n.‡ If KMN = 1, these Y's do not exist. if KMN = 2, they do exist. The three statements after statement 26 set

$$\mathrm{W9} = W_9 \tag{3.232}$$

$$\mathrm{FM1B} = \frac{mc}{b} \tag{3.233}$$

$$\mathrm{TB} = \frac{2\pi}{(ka)(k_{mn}b)} \tag{3.234}$$

where W_9 is given by eq. (3.7) of [2]. The two statements before DO loop 24 appear in the second group of extracted FORTRAN statements in Section 3.4.5.

†Here, (1.6) and (1.7) with p and q replaced by i and j are meant.

‡No mention is made of $Y_{ij}^{3,1\mathrm{TM},2\mathrm{TE}}$ and $Y_{ij}^{3,2\mathrm{TM},2\mathrm{TE}}$ because they are not computed. Instead, they are obtained from (3.67).

3.4.8 Calculations Involving r, s, n, q, m, and p

In view of (3.73), (3.74), (3.86), (3.80), (3.87), and (3.81), calculations involving r, s, n, q, m, and p are calculations involving R, S, N, Q, M, and P. As indicated in the arrangement of DO loops shown in Section 3.4.2, these calculations occur in DO loop 24 but not in DO loop 28. They are performed by all statements in DO loop 24 prior to entry into DO loop 28.

Before statement 27 is executed,

$$\text{KPQ} = \begin{cases} 1, & q = 0 \text{ or } p = 0 \\ 2, & \text{otherwise.} \end{cases} \tag{3.235}$$

Given p and q, KPQ indicates whether $\underline{M}_{pq}^{1\text{TM}}$ of (1.6) exists. Since the subscript j in eq. (3.2) of [2] is a condensation of the subscripts p and q, KPQ indicates whether $Y_{ij}^{3,1\text{TE},1\text{TM}}$ and $Y_{ij}^{3,2\text{TE},1\text{TM}}$ exist, given the values of p and q. If KPQ = 1, these Y's do not exist. If KPQ = 2, they do exist.

The seventh through seventeenth statements in DO loop 24 set

$$\text{W10} = W_{10} \tag{3.236}$$
$$\text{W11} = W_{11} \tag{3.237}$$
$$\text{T1} = \hat{T} \tag{3.238}$$
$$\text{PH1P} = \phi_p^{(1)} \tag{3.239}$$
$$\text{PH2P} = \phi_p^{(2)} \tag{3.240}$$
$$\text{PH3P} = \phi_p^{(3)} \tag{3.241}$$
$$\text{PH4P} = \phi_p^{(4)}. \tag{3.242}$$

In (3.236)–(3.238), W_{10}, W_{11}, and $\hat{T}$ are given, respectively, by eqs. (3.8), (3.9), and (3.5) of [2]. As defined by the eighteenth statement in DO loop 24, KB indicates whether $Y_{ij}^{3,1\text{TM},1\text{TM}}$ and $Y_{ij}^{3,2\text{TM},1\text{TM}}$ exist. If KB = 1, these Y's do not exist. If KB = 2, they do exist. The nineteenth through twenty-fifth statements in DO loop 24 set KMM, KEM, KME, and KEE so that

$$\text{Y(KMM)} = -j\eta Y_{ij}^{3,1\text{TM},1\text{TM}} \tag{3.243}$$
$$\text{Y(KEM)} = -j\eta Y_{ij}^{3,1\text{TE},1\text{TM}} \tag{3.244}$$
$$\text{Y(KME)} = -j\eta Y_{ij}^{3,1\text{TM},1\text{TE}} \tag{3.245}$$
$$\text{Y(KEE)} = -j\eta Y_{ij}^{3,1\text{TE},1\text{TE}}. \tag{3.246}$$

At this point in the development, equality has not yet been obtained in the four equations above. Presently, the equal sign in each of these equations means that the contribution to the right-hand side of the equation due to the rs^{th} term of the appropriate double sum in eqs. (3.1)–(3.4) of [2] will be added to what was previously stored in the computer program variable on the left-hand side of the equation. In (3.243)–(3.246),

$$\text{KMM} = (\text{PTM} - 1) * \text{K2} + \text{MTM} \tag{3.247}$$

$$\text{KEM} = (\text{PTM} - 1) * \text{K2} + \text{KTM} + \text{MTE} \tag{3.248}$$

$$\text{KME} = (\text{KTM} + \text{PTE} - 1) * \text{K2} + \text{MTM} \tag{3.249}$$

$$\text{KEE} = (\text{KTM} + \text{PTE} - 1) * \text{K2} + \text{KTM} + \text{MTE}. \tag{3.250}$$

The variables K and MJ defined in the two statements prior to DO loop 28 will be treated in the next section.

3.4.9 Calculations Involving r, s, n, q, m, p, and α

In view of (3.73), (3.74), (3.86), (3.80), (3.87), (3.81), and (3.75), calculations involving r, s, n, q, m, p, and α are calculations involving R, S, N, Q, M, P, and J. As indicated in the arrangement of DO loops shown in Section 3.4.2, these calculations occur in DO loop 28.

By virtue of the two statements prior to DO loop 28, statement 57, and the statement after statement 57, we have

$$\text{K} = (\text{J} - 1) * \text{K1} \tag{3.251}$$

$$\text{MJ} = \text{M} + (\text{J} - 1) * \text{PMAX} \tag{3.252}$$

in DO loop 28. The first four statements in DO loop 28 set

$$\text{PH1MJ} = \phi^{\alpha 1 \gamma 1} \tag{3.253}$$

$$\text{PH2MJ} = \phi^{\alpha 1 \gamma 2} \tag{3.254}$$

$$\text{PH3MJ} = \phi^{\alpha 2 \gamma 1} \tag{3.255}$$

$$\text{PH4MJ} = \phi^{\alpha 2 \gamma 2} \tag{3.256}$$

where $\alpha = \text{J}$ and $\gamma = 1$. Furthermore, the ϕ's are given by eqs. (3.32)–(3.39) of [2]. The fifth through eleventh statements in DO loop 28 set

$$\text{PAG1} = \hat{\phi}^{\alpha \gamma 1} \tag{3.257}$$

$$\mathrm{PAG2} = \hat{\phi}^{\alpha\gamma 2} \tag{3.258}$$
$$\mathrm{PAG3} = \hat{\phi}^{\alpha\gamma 3} \tag{3.259}$$
$$\mathrm{PAG4} = \hat{\phi}^{\alpha\gamma 4} \tag{3.260}$$

where $\alpha = \mathrm{J}$ and $\gamma = 1$. Moreover, the $\hat{\phi}$'s are given by eqs. (3.28)–(3.31) of [2].

If either $x_{rs} < ka$ or $x'_{rs} < ka$, then statement 30 allows execution to continue to statement 50. The block of statements beginning with statement 50 and ending with the second statement before statement 56 performs the following four operations:

1. It adds to Y(IMM) the rs term of the sum in eq. (3.1) of [2].
2. It adds to Y(IEM) the rs term of the sum in eq. (3.2) of [2].
3. It adds to Y(IME) the rs term of the sum in eq. (3.3) of [2].
4. It adds to Y(IEE) the rs term of the sum in eq. (3.4) of [2].

In the four items above, $\alpha = \mathrm{J}$ and $\gamma = 1$ in eqs. (3.1)–(3.4) of [2]. As defined by statements 38, 39, 40, and 35, respectively,

$$\mathrm{IMM} = \mathrm{KMM} + K \tag{3.261}$$
$$\mathrm{IEM} = \mathrm{KEM} + K \tag{3.262}$$
$$\mathrm{IME} = \mathrm{KME} + K \tag{3.263}$$
$$\mathrm{IEE} = \mathrm{KEE} + K \tag{3.264}$$

where K is given by (3.251). Furthermore, KMN, KEM, KME, and KEE are given by (3.247)–(3.250), respectively.

Statement 50 and the three statements following it set

$$\mathrm{S1} = \hat{S}_1 \tag{3.265}$$
$$\mathrm{S3} = \hat{S}_3 \tag{3.266}$$
$$\mathrm{S4} = \hat{S}_4 \tag{3.267}$$
$$\mathrm{S5} = \hat{S}_5 \tag{3.268}$$

where the $\hat{S}$'s are given, respectively, by eqs. (3.10)–(3.13) of [2]. The statement after statement 38 sets

$$\mathrm{YMM} = \hat{T}(W_8\hat{S}_1 + W_9\hat{S}_3 - W_{10}\hat{S}_4 - W_{11}\hat{S}_5) \tag{3.269}$$

where the right-hand side of (3.269) appears in eq. (3.1) of [2]. The statement after statement 39 sets

$$\text{YEM} = \hat{T}(W_9\hat{S}_1 - W_8\hat{S}_3 - W_{11}\hat{S}_4 + W_{10}\hat{S}_5). \tag{3.270}$$

The statement after statement 40 sets

$$\text{YME} = \hat{T}(W_{10}\hat{S}_1 + W_{11}\hat{S}_3 + W_8\hat{S}_4 + W_9\hat{S}_5). \tag{3.271}$$

The statement after statement 35 sets

$$\text{YEE} = \hat{T}(W_{11}\hat{S}_1 - W_{10}\hat{S}_3 + W_9\hat{S}_4 - W_8\hat{S}_5). \tag{3.272}$$

If $x_{rs} \geq ka$ and $x'_{rs} \geq ka$, then control statement 30 sends execution to statement 56. The block of statements beginning with statement 56 and ending with the statement before statement 57 does for the case where $x_{rs} \geq ka$ and $x'_{rs} \geq ka$ what is done for the case where either $x_{rs} < ka$ or $x'_{rs} < ka$ by the block of statements beginning with statement 50 and ending with the second statement before statement 56. The former block of statements, which is executed more and more times as XM is made larger and larger, executes faster than the latter block of statements. The real variables S1R, S3R, S4R, S5R, Z1R, Z2R, Z3R, Z4R, and Z5R are easier to deal with than their complex counterparts S1, S3, S4, S5, Z1, Z2, Z3, Z4, and Z5.

If a normal exit from DO loop 19 occurs, then the stop statement after statement 44 terminates execution. This termination can be avoided by either decreasing XM or replacing the "500" in the statement

DO 19 R=1, 500

by a larger number. If the sixth statement in DO loop 19 sends control to statement 25 when R = 1, then the stop statement after statement 42 terminates execution. This termination can be avoided by increasing XM.

3.5 The Matrix Equation and Its Solution

In Section 3.5.1, the product of $-j\eta$ with the admittance matrix $[Y^1+Y^2+Y^3]$ in the matrix equation (1.32) is put in the one-dimensional array Y. Then, in Section 3.5.1, using the values of the elements of $-j\vec{I}^{1\text{TM}}e^{j\beta_{01}^{\text{TM}}L_3}$, $-j\vec{I}^{1\text{TE}}$

$\cdot e^{j\beta_{01}^{TM}L_3}$, $-j\vec{I}^{2TM}e^{j\beta_{01}^{TM}L_3}$, and $-j\vec{I}^{2TE}e^{j\beta_{01}^{TM}L_3}$ stored in TI, the matrix equation (1.32) is solved for the elements of $\frac{1}{\eta}\vec{V}^{1TM}e^{j\beta_{01}^{TM}L_3}$, $\frac{1}{\eta}\vec{V}^{1TE}e^{j\beta_{01}^{TM}L_3}$, $\frac{1}{\eta}\vec{V}^{2TM}$ $\cdot e^{j\beta_{01}^{TM}L_3}$, and $\frac{1}{\eta}\vec{V}^{2TE}e^{j\beta_{01}^{TM}L_3}$.

3.5.1 The Admittance Matrix

Nested DO loops 89 and 88 implement (3.67). The fifth statement in DO loop 88 sets

$$-j\eta\begin{bmatrix} Y^{3,1TM,2TM} & Y^{3,1TM,2TE} \\ Y^{3,1TE,2TM} & Y^{3,1TE,2TE} \end{bmatrix}_{IJ} = -j\eta\begin{bmatrix} Y^{3,2TM,1TM} & Y^{3,2TM,1TE} \\ Y^{3,2TE,1TM} & Y^{3,2TE,1TE} \end{bmatrix}_{IJ} \tag{3.273}$$

where the subscript IJ denotes the IJ[th] element of the matrix enclosed in the square brackets. The sixth statement in DO loop 88 sets

$$-j\eta\begin{bmatrix} Y^{3,2TM,2TM} & Y^{3,2TM,2TE} \\ Y^{3,2TE,2TM} & Y^{3,2TE,2TE} \end{bmatrix}_{IJ} = -j\eta\begin{bmatrix} Y^{3,1TM,1TM} & Y^{3,1TM,1TE} \\ Y^{3,1TE,1TM} & Y^{3,1TE,1TE} \end{bmatrix}_{IJ}. \tag{3.274}$$

After exit from nested DO loops 89 and 88, the elements of $-j\eta Y^3$ will be stored by columns in the one-dimensional array Y. Here, Y^3 is given by (1.38).

Since $-j\eta\,[Y^1 + Y^2]$ is a diagonal matrix whose nonzero elements reside in the one-dimensional array YREC, the elements of $-j\eta\,[Y^1 + Y^2 + Y^3]$ can be put in the one-dimensional array Y by adding the entries of YREC to the "diagonal" entries of Y.[†] This addition is done in DO loop 43.

3.5.2 Solution of the Matrix Equation

The matrix equation (1.32) is solved by means of the two statements after statement 43. These two statements put in V the elements of the solution $\vec{V}$ of the matrix equation

$$Y\vec{V} = \vec{I} \tag{3.275}$$

when Y is the K2 × K2 matrix whose elements are stored by columns in the one-dimensional array Y and $\vec{I}$ is the K2 × 1 column vector whose elements reside in TI.[‡]

[†]The "diagonal" entries of Y are the the entries of Y that contain the diagonal elements of the matrix. Thus, the "diagonal entries" of Y are Y(1), Y(K2 + 2), Y(2∗K2 + 3), ⋯ Y ((K2)²).

[‡]The subroutines DECOMP and SOLVE are listed in Chapter 11.

The elements of $-j\eta\,[Y^1 + Y^2 + Y^3]$ were, as described in Section 3.5, stored by columns in the one-dimensional array Y. Moreover, the elements of $-j\vec{I}^{1\mathrm{TM}}e^{j\beta_{01}^{\mathrm{TM}}L_3}$, $-j\vec{I}^{1\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}$, $-j\vec{I}^{2\mathrm{TM}}e^{j\beta_{01}^{\mathrm{TM}}L_3}$, and $-j\vec{I}^{2\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}$, were put in TI by nested DO loops 29 and 52 whose indices appear in (3.148) and (3.149). Thus, the two statements after statement 43 put in V the elements of $\frac{1}{\eta}\vec{V}^{1\mathrm{TM}}e^{j\beta_{01}^{\mathrm{TM}}L_3}$, $\frac{1}{\eta}\vec{V}^{1\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}$, $\frac{1}{\eta}\vec{V}^{2\mathrm{TM}}e^{j\beta_{01}^{\mathrm{TM}}L_3}$, and $\frac{1}{\eta}\vec{V}^{2\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}$ where $\vec{V}^{1\mathrm{TM}}$, $\vec{V}^{1\mathrm{TE}}$, $\vec{V}^{2\mathrm{TM}}$, and $\vec{V}^{2\mathrm{TE}}$ are the column vectors that satisfy (1.32).

3.6 The Coefficients of the TE_{10} Modes in the Rectangular Waveguides

The normalized complex coefficient of the $\mp x$-traveling TE_{10} wave in the left-hand rectangular waveguide of Fig. 2 is called $C_{10}^{1\mathrm{TE}\mp}$. Here, "$\mp x$-traveling" means traveling in the $\mp x$-direction. The normalization of $C_{10}^{1\mathrm{TE}\mp}$ is specified by eq. (5.30) of [2]. The right-hand side of this equation is the electric field that would exist in the circular waveguide if the transverse electric field of the z-traveling TM_{01}^{e} wave at $z = L_3$ in the circular waveguide were $\sqrt{Z_{01}^{\mathrm{TM}eo}}\underline{e}_{01}^{\mathrm{TM}e}$ where $Z_{01}^{\mathrm{TM}eo}$ and $\underline{e}_{01}^{\mathrm{TM}e}$ are, respectively, the characteristic impedance and transverse electric vector of the TM_{01}^{e} mode field defined by eq. (B.1) of [1]. According to eq. (A.15) of [1], the quantity that multiplies $C_{10}^{1\mathrm{TE}-}$ on the right-hand side of eq. (5.30) of [2] is $\underline{e}_{10}^{\mathrm{TE}}/\sqrt{Y_{10}^{\mathrm{TE}}}$ when $x = -x_o$. According to eq. (A.14) of [1], the quantity that multiplies $C_{10}^{1\mathrm{TE}+}$ on the right-hand side of eq. (5.30) of [2] is $\underline{e}_{10}^{\mathrm{TE}}/\sqrt{Y_{10}^{\mathrm{TE}}}$ when $x = -x_o$. Now, the time-average power of the z-traveling TM_{01}^{e} wave whose transverse electric field is $\sqrt{Z_{01}^{\mathrm{TM}eo}}\underline{e}_{01}^{\mathrm{TM}e}$ at $z = L_3$ in the circular waveguide is unity. The time-average power of the $\mp x$-traveling TE_{10} wave whose electric field is $\underline{e}_{10}^{\mathrm{TE}}/\sqrt{Y_{10}^{\mathrm{TE}}}$ when $x = x_o$ in the left-hand rectangular waveguide is also unity. Hence, $\left|C_{10}^{1\mathrm{TE}\mp}\right|^2$ is the ratio of the time-average power of the $\mp x$-traveling TE_{10} wave in the left-hand rectangular waveguide to the time-average power of the z-traveling TM_{01}^{e} wave in the circular waveguide.

The normalized complex coefficient of the $\pm x$-traveling TE_{10} wave in the right-hand rectangular waveguide of Fig. 2 is called $C_{10}^{2\mathrm{TE}\pm}$. The normalization of $C_{10}^{2\mathrm{TE}\pm}$ is specified by eq. (5.31) of [2]. Proceeding as in the previous paragraph, we see that $\left|C_{10}^{2\mathrm{TE}\pm}\right|^2$ is the ratio of the time-average power of the $\pm x$-traveling TE_{10} wave in the right-hand rectangular waveguide to the time-average power of the z-traveling TM_{01}^{e} wave in the circular waveguide.

3.6.1 The Modal Coefficients in the Left-Hand Rectangular Waveguide

The modal coefficients $C_{10}^{1\mathrm{TE}-}$ and $C_{10}^{1\mathrm{TE}+}$ are given, respectively, by eqs. (5.32) and (5.33) of [2]. The block of statements beginning with the statement after statement 206 and ending with statement 69 puts these coefficients in C1OUT and C1IN and then writes them out. In this block of statements,

$$\mathrm{BET} = \beta_{10} a \tag{3.276}$$

$$\mathrm{ARG} = \beta_{10} x_1 \tag{3.277}$$

$$\mathrm{CS} = \cos(\beta_{10} x_1) \tag{3.278}$$

$$\mathrm{SN} = \sin(\beta_{10} x_1) \tag{3.279}$$

$$\mathrm{S1} = \tfrac{1}{2}\sqrt{\frac{\beta_{10}}{\beta_{01}^{\mathrm{TM}}}} \tag{3.280}$$

$$\mathrm{V(KV)} = \tfrac{1}{\eta} V_{10}^{1\mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3} \tag{3.281}$$

$$\mathrm{SA} = \sqrt{\frac{\beta_{10}}{\beta_{01}^{\mathrm{TM}}}}\left(\frac{1}{2\left(Z_1 Y_{10}^{\mathrm{TE}}\cos(\beta_{10}x_1) + j\sin(\beta_{10}x_1)\right)}\right)\left(\tfrac{1}{\eta} V_{10}^{1\mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3}\right) \tag{3.282}$$

$$\mathrm{C1OUT} = C_{10}^{1\mathrm{TE}-} \tag{3.283}$$

$$\mathrm{C1IN} = C_{10}^{1\mathrm{TE}+} \tag{3.284}$$

where β_{10}, x_1, $C_{10}^{1\mathrm{TE}-}$, and $C_{10}^{1\mathrm{TE}+}$ are given, respectively, by eqs. (2.5), (5.12), (5.32) and (5.33) of [2]. In (3.281), $V_{10}^{1\mathrm{TE}}$ is the first element of $\vec{V}^{1\mathrm{TE}}$. Here, the subscript "10" is the same as that in TE_{10}. The TE_{10} mode is the first TE mode. Now, $\frac{1}{\eta} V_{10}^{1\mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3}$ resides in V(KTM + 1) because the KTM elements of $\frac{1}{\eta}\vec{V}^{1\mathrm{TM}} e^{j\beta_{01}^{\mathrm{TM}} L_3}$ precede $\frac{1}{\eta} V_{10}^{1\mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3}$ in V.

3.6.2 The Modal Coefficients in the Right-Hand Rectangular Waveguide

The modal coefficients $C_{10}^{2\text{TE}+}$ and $C_{10}^{2\text{TE}-}$ are given, respectively, by eqs. (5.36) and (5.37) of [2].[†] The block of statements beginning with the statement after statement 69 and ending with statement 70 puts these coefficients in C2OUT and C2IN and writes them out. In this block of statements,

$$\text{ARG} = \beta_{10}x_2 \tag{3.285}$$

$$\text{CS} = \cos(\beta_{10}x_2) \tag{3.286}$$

$$\text{SN} = \sin(\beta_{10}x_2) \tag{3.287}$$

$$\text{V(KV)} = \frac{1}{\eta}V_{10}^{2\text{TE}}e^{j\beta_{01}^{\text{TM}}L_3} \tag{3.288}$$

$$\text{SA} = \sqrt{\frac{\beta_{10}}{\beta_{01}^{\text{TM}}}}\left(\frac{1}{2\left(Z_2Y_{10}^{\text{TE}}\cos(\beta_{10}x_2) + j\sin(\beta_{10}x_2)\right)}\right)\left(\frac{1}{\eta}V_{10}^{2\text{TE}}e^{j\beta_{01}^{\text{TM}}L_3}\right) \tag{3.289}$$

$$\text{C2OUT} = C_{10}^{2\text{TE}+} \tag{3.290}$$

$$\text{C2IN} = C_{10}^{2\text{TE}-} \tag{3.291}$$

where x_2, $C_{10}^{2\text{TE}+}$, and $C_{10}^{\text{TE}-}$ are given, respectively, by eqs. (5.21), (5.36), and (5.37) of [2]. In (3.288), $V_{10}^{2\text{TE}}$ is the first element of $\vec{V}^{2\text{TE}}$. Now, $\frac{1}{\eta}V_{10}^{2\text{TE}}e^{j\beta_{01}^{\text{TM}}L_3}$ resides in V(K1 + KTM + 1) because the KTM elements of $\frac{1}{\eta}\vec{V}^{1\text{TM}}e^{j\beta_{01}^{\text{TM}}L_3}$, the KTE elements of $\frac{1}{\eta}\vec{V}^{1\text{TE}}e^{j\beta_{01}^{\text{TM}}L_3}$, and the KTM elements of $\frac{1}{\eta}\vec{V}^{2\text{TM}}e^{j\beta_{01}^{\text{TM}}L_3}$, precede $\frac{1}{\eta}V_{10}^{2\text{TE}}e^{j\beta_{01}^{\text{TM}}L_3}$ in V.

3.7 The Coefficients of the Propagating Modes in the Circular Waveguide

The normalized complex coefficients of the $-z$-traveling TM_{01}^{e}, TE_{11}^{e}, and TE_{11}^{o} waves in the circular waveguide are, respectively, $C_{01}^{\text{TM}e}$, $C_{11}^{\text{TE}e}$, and $C_{11}^{\text{TE}o}$ of eqs. (6.88), (6.96), and (6.99) of [2]. The normalization of $C_{01}^{\text{TM}e}$,

[†]There is a misprint in eq. (5.37) of [2]. The left-hand side of eq. (5.37) of [2] should be $C_{10}^{2\text{TE}-}$.

$C_{11}^{\mathrm{TE}e}$, and $C_{11}^{\mathrm{TE}o}$ is specified by eq. (6.75) of [2]. The right-hand side of this equation is the electric field that would exist in the circular waveguide if the transverse electric field of the z-traveling TM_{01}^{e} wave at $z = L_3$ in the circular waveguide were $\sqrt{Z_{01}^{\mathrm{TM}eo}}\underline{e}_{01}^{\mathrm{TM}e}$. According to eq. (B.2) of [1], the quantity that multiplies $C_{01}^{\mathrm{TM}e}$ on the right-hand side of eq. (6.75) of [2] is $\sqrt{Z_{01}^{\mathrm{TM}eo}}\underline{e}_{01}^{\mathrm{TM}e}$ when $z = L_3$. According to eq. (B.36) of [1], the quantity that multiplies $C_{11}^{\mathrm{TE}e}$ on the right-hand side of (6.75) of [2] is $\underline{e}_{11}^{\mathrm{TE}e}/\sqrt{Y_{11}^{\mathrm{TE}eo}}$ when $z = L_3$. From eq. (B.56) of [1], the quantity that multiplies $C_{11}^{\mathrm{TE}o}$ on the right-hand side of (6.75) of [2] is $\underline{e}_{11}^{\mathrm{TE}o}/\sqrt{Y_{11}^{\mathrm{TE}eo}}$. Since the transverse electric fields $\sqrt{Z_{01}^{\mathrm{TM}eo}}\underline{e}_{01}^{\mathrm{TM}e}$, $\underline{e}_{11}^{\mathrm{TE}e}/\sqrt{Y_{11}^{\mathrm{TE}eo}}$, and $\underline{e}_{11}^{\mathrm{TE}o}/\sqrt{Y_{11}^{\mathrm{TE}eo}}$ are those of traveling waves of unit time-average power in the circular waveguide, it follows that $|C_{01}^{\mathrm{TM}e}|^2$, $|C_{11}^{\mathrm{TE}e}|^2$, and $|C_{11}^{\mathrm{TE}o}|^2$ are, respectively, the ratio of the time-average power of the $-z$-traveling TM_{01}^{e} wave, that of the $-z$-traveling TE_{11}^{e} wave, and that of the $-z$-traveling TE_{11}^{o} wave to the time-average power of the z-traveling TM_{01}^{e} wave.

DO loop 55 accumulates $C_{01}^{\mathrm{TM}e}+1$, $C_{11}^{\mathrm{TE}e}$, and $C_{11}^{\mathrm{TE}o}$ in CTME, CTEE, and CTEO, respectively. Here, $C_{01}^{\mathrm{TM}e}$, $C_{11}^{\mathrm{TE}e}$, and $C_{11}^{\mathrm{TE}o}$ are given, respectively, by eqs. (6.88), (6.96), and (6.99) of [2]. After execution of the second statement after statement 55, we will have

$$\mathrm{CTME} = C_{01}^{\mathrm{TM}e} \tag{3.292}$$

$$\mathrm{CTEE} = C_{11}^{\mathrm{TE}e} \tag{3.293}$$

$$\mathrm{CTEO} = C_{11}^{\mathrm{TE}o} \tag{3.294}$$

$$\mathrm{CTMMS} = |C_{01}^{\mathrm{TM}e} + 1|^2. \tag{3.295}$$

In (3.295), $C_{01}^{\mathrm{TM}e}+1$ is the contribution to $C_{01}^{\mathrm{TM}e}$ due to the magnetic currents $-\underline{M}^{(1)}$ and $-\underline{M}^{(2)}$ placed on the inside surfaces of the conductors that close the apertures in Fig. 2. This contribution was calculated for use in Section 3.8.2.

3.8 The Time-Average Power of the Propagating Modes in the Waveguides

The ratio of the time-average power of any propagating mode to the time-average power of the incident TM_{01}^e mode is the square of the magnitude of the normalized complex coefficient of the propagating mode. In addition to the z-traveling TM_{01}^e wave in the circular waveguide, there are seven propagating modes:

1. The $-x$-traveling TE_{10} wave in the left-hand rectangular waveguide.
2. The x-traveling TE_{10} wave in the left-hand rectangular waveguide.
3. The x-traveling TE_{10} wave in the right-hand rectangular waveguide.
4. The $-x$-traveling TE_{10} wave in the right-hand rectangular waveguide.
5. The $-z$-traveling TM_{01}^e wave in the circular waveguide.
6. The $-z$-traveling TE_{11}^e wave in the circular waveguide.
7. The $-z$-traveling TE_{11}^o wave in the circular waveguide.

Each time-average power mentioned in Sections 3.8.1, 3.8.2, 3.8.3, 3.9, 3.9.1, and 3.9.2 to follow is understood to be relative to the time-average power of the z-traveling TM_{01}^e wave in the circular waveguide. In other words, each of these time-average powers is tacitly assumed to be the ratio of that to the time-average power of the incident TM_{01}^e wave in the circular waveguide.

3.8.1 The Time-Average Power of the Propagating Modes in the Rectangular Waveguides

The time-average power of each propagating mode in the rectangular waveguides appears in eq. (5.49) of [2] for the time-average power P_t transmitted into the rectangular waveguides. The five statements after statement 81 set

$$\mathrm{C1OUTS} = |C_{10}^{1\mathrm{TE}-}|^2 \tag{3.296}$$

$$\mathrm{C1INS} = |C_{10}^{1\mathrm{TE}+}|^2 \tag{3.297}$$

$$\mathrm{C2OUTS} = |C_{10}^{2\mathrm{TE}+}|^2 \tag{3.298}$$

$$\mathrm{C2INS} = |C_{10}^{2\mathrm{TE}-}|^2 \tag{3.299}$$

$$\mathrm{PT} = P_t. \tag{3.300}$$

The time-average power P_t transmitted into the rectangular waveguides is the time-average power radiated by $\underline{M}^{(1)}$ and $\underline{M}^{(2)}$ in the rectangular waveguides because there are no other sources therein.

3.8.2 The Time-Average Power of the Propagating Modes in the Circular Waveguide

The time-average power of each propagating mode in the circular waveguide appears in eq. (8.1) of [2] for the time-average power P_r reflected in the circular waveguide. The four statements after statement 82 set

$$\text{CTMES} = |C_{01}^{\text{TMe}}|^2 \tag{3.301}$$

$$\text{CTEES} = |C_{11}^{\text{TEe}}|^2 \tag{3.302}$$

$$\text{CTEOS} = |C_{11}^{\text{TEo}}|^2 \tag{3.303}$$

$$\text{PR} = P_r. \tag{3.304}$$

If the magnetic currents $-\underline{M}_1$ and $-\underline{M}_2$ were somehow forced to flow in the circular waveguide in the absence of the incident TM_{01}^e mode due to $\underline{J}^{\text{imp}}$, then the total time-average power of the propagating modes in the circular waveguide would be P_{rm} given by

$$P_{rm} = |C_{01}^{\text{TMe}} + 1|^2 + |C_{11}^{\text{TEe}}|^2 + |C_{11}^{\text{TEo}}|^2. \tag{3.305}$$

The subscript "rm" on P_{rm} indicates that it is the time-average reflected power due to the magnetic currents. The fifth statement after statement 82 sets

$$\text{PRM} = P_{rm}. \tag{3.306}$$

The above propagating mode power P_{rm} is the time-average power that the magnetic currents $-\underline{M}_1$ and $-\underline{M}_2$ would radiate if they existed alone in the circular waveguide.

The three statements after statement 90 set

$$\text{BKAPLT(KA)} = ka \tag{3.307}$$

$$\text{PTRAN(KA)} = P_t \tag{3.308}$$

$$\text{PREFL(KA)} = P_r. \tag{3.309}$$

The one-dimensional arrays BKAPLT, PTRAN, and PREFL will be written out after exit from DO loop 48.

3.8.3 Conservation of Power

The sum of the time-average power P_t transmitted into the rectangular waveguides and the time-average power P_r reflected in the circular waveguide should be equal to the time-average power of the z-traveling TM_{01}^{e} wave in the circular waveguide. Since P_t and P_r are meant to be ratios of time-average powers to the time-average power of the z-traveling TM_{01}^{e} wave, we should have

$$P_t + P_r = 1. \tag{3.310}$$

The fourth statement after statement 90 sets

$$\mathrm{PTOTAL} = P_t + P_r. \tag{3.311}$$

In the sample output of Section 2.2.2, PTOTAL is indeed unity. This verifies the calculated values of P_t and P_r. Other verifications are given in Sections 3.9.1 and 3.9.2.

3.9 The Time-Average Power Radiated by the Magnetic Currents

In this section, we assume that $\underline{J}^{\mathrm{imp}}$ is absent so that there is no z-traveling TM_{01}^{e} wave in Fig. 2. However, we assume that the magnetic currents $\pm\underline{M}^{(1)}$ and $\pm\underline{M}^{(2)}$ are somehow forced to flow. The time-average power radiated by the magnetic currents $\underline{M}^{(1)}$ and $\underline{M}^{(2)}$ in the rectangular waveguides was expressed in terms of the time-average power of the propagating modes as indicated by (3.300) where P_t is given by eq. (5.49) of [2]. The time-average power radiated by the magnetic currents $-\underline{M}^{(1)}$ and $-\underline{M}^{(2)}$ alone in the circular waveguide was expressed in terms of the time-average power of the propagating modes as P_{rm} of (3.05).

Alternatively, the time-average power P radiated by a magnetic current $\underline{M}$ on a surface S is given by

$$P = -\mathrm{Re}\left\{\frac{k}{\eta\beta_{01}^{\mathrm{TM}}}\iint_S \underline{M}^* \cdot \underline{H}(\underline{0},\underline{M})\,ds\right\} \tag{3.312}$$

where $\underline{M}^*$ is the complex conjugate of $\underline{M}$, $\underline{H}$ is the magnetic field due to $\underline{M}$, ds is the differential element of surface area, and "Re" denotes the real part.† The "$\underline{0}$" in $\underline{H}(\underline{0}, \underline{M})$ indicates that there is no electric current source. The factor $k/(\eta\beta_{01}^{\mathrm{TM}})$ is due to the implicit normalization of P. This factor is the reciprocal of the time-average power of the z-traveling TM_{01}^{e} wave. It normalizes P so that P is the time-average power that would result if the time-average power of the z-traveling TM_{01}^{e} wave were unity.

3.9.1 The Time-Average Power Radiated by the Magnetic Currents in the Rectangular Waveguides

In this section, (3.312) will be used to obtain an alternate expression for the time-average power radiated by the magnetic currents $\underline{M}^{(1)}$ and $\underline{M}^{(2)}$ in the rectangular waveguides.

Relabeling the V's and the $\underline{M}$'s in (1.6) $\{V_1, V_2, V_3, \cdots, V_{\mathrm{K1}}\}$ and $\{\underline{M}_1, \underline{M}_2, \underline{M}_3, \cdots, \underline{M}_{\mathrm{K1}}\}$ and relabeling the V's and the $\underline{M}$'s in (1.7) $\{V_{\mathrm{K1}+1}, V_{\mathrm{K1}+2}, \cdots, V_{\mathrm{K2}}\}$ and $\{\underline{M}_{\mathrm{K1}+1}, \underline{M}_{\mathrm{K1}+2}, \cdots, \underline{M}_{\mathrm{K2}}\}$ where K1 and K2 are given by (3.10) and (3.52), we express the combination of $\underline{M}^{(1)}$ and $\underline{M}^{(2)}$ of (1.6) and (1.7) as the single magnetic current $\underline{M}$ given by

$$\underline{M} = \sum_{j=1}^{\mathrm{K2}} V_j \underline{M}_j. \tag{3.313}$$

Since $\underline{M}_j$ is real, substitution of (3.313) into (3.312) gives

$$P_{ta} = \mathrm{Re}\left\{ \frac{k}{\eta\beta_{01}^{\mathrm{TM}}} \sum_{i=1}^{\mathrm{K2}} V_i^* \sum_{j=1}^{\mathrm{K2}} \left[Y^1 + Y^2\right]_{ij} V_j \right\} \tag{3.314}$$

where the subscript "ta" on P_{ta} indicates that P_{ta} is the transmitted power calculated by means of the alternative method whereby (3.312) is used. In (3.314),

$$[Y^1 + Y^2]_{ij} = -\iint \underline{M}_i \cdot \underline{H}(\underline{0}, \underline{M}_j)\, ds \tag{3.315}$$

†In our "root-mean-square" notation, a complex phasor such as $\underline{M}$ indicates the time-dependent quantity $\sqrt{2}\mathrm{Re}(\underline{M}e^{j\omega t})$. If "peak-value" notation were used, $\underline{M}$ would indicate the time-dependent quantity $\mathrm{Re}(\underline{M}e^{j\omega t})$ and the right-hand side of (3.312) would be divided by 2.

where the integration is over A_1 if $i \leq$ K1 and over A_2 if $i >$ K1. If $j \leq$ K1, then $\underline{H}(\underline{0}, \underline{M}_j)$ is the magnetic field due to $\underline{M}_j$ in the left-hand rectangular waveguide. If $j >$ K1, then $\underline{H}(\underline{0}, \underline{M}_j)$ is the magnetic field due to $\underline{M}_j$ in the right-hand rectangular waveguide. The "$\underline{0}$" in $\underline{H}(\underline{0}, \underline{M}_j)$ indicates that there is no electric current source.

Now, the matrix $[Y^1 + Y^2]$ in (3.315) is the same as that in (1.32). As stated in Section 3.3, this matrix is a diagonal matrix, and the product of its ii^{th} element with $-j\eta$ resides in YREC(i). Furthermore, $\frac{1}{\eta} V_i e^{j\beta_{01}^{\text{TM}} L_3}$ resides in V(i). Hence, we recast (3.314) as

$$P_{ta} = -\frac{k}{\beta_{01}^{\text{TM}}} \text{Im} \sum_{i=1}^{\text{K2}} \left\{ -j\eta [Y^1 + Y^2]_{ii} \left(\tfrac{1}{\eta} V_i e^{j\beta_{01}^{\text{TM}} L_3} \right)^* \left(\tfrac{1}{\eta} V_i e^{j\beta_{01}^{\text{TM}} L_3} \right) \right\} \quad (3.316)$$

where "Im" denotes the imaginary part. The second statement in DO loop 91 accumulates in YMM the summation in (3.316). The two statements after statement 91 set

$$\text{BKAB} = \frac{k}{\beta_{01}^{\text{TM}}} \quad (3.317)$$

$$\text{PTA} = P_{ta}. \quad (3.318)$$

Now, P_t of (3.300) should be equal to P_{ta} of (3.318). Hence, the computer program variables PT and PTA should be equal to each other. In the output listed in Section 2.2.2, they differ by two units in the seventh significant figure. This discrepancy can be attributed to roundoff error because not all calculations were done in double precision. Thus, the computed value of P_t is verified.

3.9.2 The Time-Average Power Radiated by the Magnetic Currents in the Circular Waveguide

In this section, (3.312) will be used to obtain an alternative expression for the time-average power that the magnetic currents $-\underline{M}^{(1)}$ and $-\underline{M}^{(2)}$ would radiate if they were the only sources in the circular waveguide.

The combination of $-\underline{M}^{(1)}$ and $-\underline{M}^{(2)}$ is expressed as the single magnetic current $\underline{M}$ given, similar to (3.313), by

$$\underline{M} = -\sum_{j=1}^{\text{K2}} V_j \underline{M}_j. \quad (3.319)$$

Substitution of (3.319) into (3.312) gives

$$P_{rma} = \frac{k}{\eta \beta_{01}^{\mathrm{TM}}} \mathrm{Re} \left\{ \sum_{i=1}^{\mathrm{K2}} V_i^* \sum_{j=1}^{\mathrm{K2}} Y_{ij}^3 V_j \right\} \tag{3.320}$$

where the subscript "*rma*" on P_{rma} indicates that P_{rma} is the reflected power due to the magnetic current calculated by means of the alternative method whereby (3.312) is used. In (3.320),

$$Y_{ij}^3 = -\iint \underline{M}_i \cdot \underline{H}^{(3)}(\underline{0}, \underline{M}_j)\, ds \tag{3.321}$$

where the integration is over A_1 if $i \leq \mathrm{K1}$ and over A_2 if $i > \mathrm{K1}$. In (3.321), $\underline{H}_3(\underline{0}, \underline{M}_j)$ is the magnetic field due to $\underline{M}_j$ radiating in the circular waveguide.

Now, Y_{ij}^3 is the ij^{th} element of the matrix Y^3 that appears in (1.32). We recast (3.320) as

$$P_{rma} = -\frac{k}{\beta_{01}^{\mathrm{TM}}} \mathrm{Im} \sum_{i=1}^{\mathrm{K2}} \left\{ \left(\tfrac{1}{\eta} V_i e^{j\beta_{01}^{\mathrm{TM}} L_3}\right)^* \sum_{j=1}^{\mathrm{K2}} \left\{ \left(-j\eta Y_{ij}^3\right) \left(\tfrac{1}{\eta} V_j e^{j\beta_{01}^{\mathrm{TM}} L_3}\right) \right\} \right\}. \tag{3.322}$$

Replacing Y_{ij}^3 by $[Y^1 + Y^2 + Y^3]_{ij} - [Y^1 + Y^2]_{ij}$ in (3.321), we obtain

$$P_{rma} = -P_{ta} - \frac{k}{\beta_{01}^{\mathrm{TM}}} \mathrm{Im} \sum_{i=1}^{\mathrm{K2}} \left\{ \left(\tfrac{1}{\eta} V_i e^{j\beta_{01}^{\mathrm{TM}} L_3}\right)^* \left(-j I_i e^{j\beta_{01}^{\mathrm{TM}} L_3}\right) \right\} \tag{3.323}$$

where P_{ta} is given by (3.316) and

$$-j I_i e^{j\beta_{01}^{\mathrm{TM}} L_3} = \sum_{j=1}^{\mathrm{K2}} \left\{ -j\eta \left[Y^1 + Y^2 + Y^3\right]_{ij} \left(\tfrac{1}{\eta} V_j e^{j\beta_{01}^{\mathrm{TM}} L_3}\right) \right\}. \tag{3.324}$$

Noting that the right-hand side of (3.324) is the product of $-je^{j\beta_{01}^{\mathrm{TM}} L_3}$ with the i^{th} element of the left-hand side of (1.32), we see that I_i of (3.324) is the i^{th} element of the column vector on the right-hand side of (1.32). As indicated in the paragraph containing (3.146), $-j I_i e^{j\beta_{01}^{\mathrm{TM}} L_3}$ was put in TI(i).

The third statement in DO loop 91 accumulates in YME the summation in (3.323). The third statement after statement 91 sets

$$\mathrm{PRMA} = P_{rma} \tag{3.325}$$

where P_{rma} is given by (3.323). The fourth statement after statement 91 writes out PTA and PRMA.

Now, P_{rm} of (3.306) should be equal to P_{rma} of (3.325). Hence, the computer program variables PRM and PRMA should be equal to each other. In the output listed in Section 2.2.2, they differ by only one unit in the seventh significant figure. Thus, the computed value of P_{rm} is verified. The verification of P_{rm} gives some verification of P_r because P_{rm} is the contribution to P_r due to the magnetic current. Note that there are three contributions to P_r: one due to the $-z$-traveling TM_{01}^e wave, one due to the magnetic current, and one due to to the interaction between the $-z$-traveling TM_{01}^e wave and the magnetic current.

3.10 The Tangential Electric Field in the Apertures

The control statement after statement 93 either allows execution to continue on to the second statement after statement 93 or sends execution to statement 48, depending on the value of KE3(KAE). See the last paragraph in Section 2.1.1. If executed, the block of statements beginning with the second statement after statement 93 and ending with the statement before statement 48 calculates and writes out values of the ϕ- and z-components of the electric field in the left-hand and right-hand apertures. See the paragraph containing (2.31).

In DO loop 105, E3A1P(J) and E3A2P(J) are set to zero. Later,

$$\frac{E_\phi^{(A1)}(\phi_j^{(A1)},0)}{|\underline{E}_{01}^{\mathrm{TM}e+}|_{\mathrm{rms}}} \text{ and } \frac{E_\phi^{(A2)}(\phi_j^{(A2)},0)}{|\underline{E}_{01}^{\mathrm{TM}e+}|_{\mathrm{rms}}},$$

given by eq. (7.13) of [2] with $\gamma = 1$ and $\gamma = 2$,[†] will be accumulated in E3A1P(J) and E3A2P(J), respectively. Here,

$$\phi_{\mathrm{J}}^{(A1)} = \pi + \left(-1 + 2\frac{\mathrm{J}-1}{\mathrm{NPHI}-1}\right)\phi_o \tag{3.326}$$

$$\phi_{\mathrm{J}}^{(A2)} = \left(-1 + 2\frac{\mathrm{J}-1}{\mathrm{NPHI}-1}\right)\phi_o. \tag{3.327}$$

[†]There is a misprint in eqs. (7.13) and (7.14) of [2]. The quantity "β_{01}^{TE}" should be replaced by "β_{01}^{TM}" in both equations.

Substituting $\phi_J^{(A1)}$ for ϕ and x_o/a of (3.143) for $(\sin\phi_o)/\phi_o$ in eq. (7.13) of [2] with $\gamma = 1$, interchanging the order of summation, and placing upper limits on the summation indices p and q,[†] we obtain

$$\frac{E_\phi^{(A1)}(\phi_J^{(A1)},0)}{|E_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}} = -(S_a)\left(\frac{x_o}{a}\right)\sum_{q=0}^{\mathrm{NMAX}-1}\sum_{\substack{p=0\\p+q\neq 0}}^{\mathrm{MM}(q+1)-1}\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\left(\frac{1}{k_{pq}b}\right)$$
$$\cdot\left\{\left(\frac{pa}{b}\right)\left(\frac{V_{pq}^{1\mathrm{TM}}e^{j\beta_{01}^{\mathrm{TM}}L_3}}{\eta}\right)-\left(\frac{qa}{c}\right)\left(\frac{V_{pq}^{1\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}}{\eta}\right)\right\}$$
$$\cdot\sin\frac{q\pi}{2}\cos\left(p\phi_J^{(1)}\right) \tag{3.328}$$

where

$$S_a = 2\pi\sqrt{\frac{\pi b}{c}}\left(\frac{k}{\beta_{01}^{\mathrm{TM}}}\right)e^{-j\beta_{01}^{\mathrm{TM}}L_3} \tag{3.329}$$

$$\phi_J^{(1)} = \pi - \frac{(\mathrm{J}-1)\pi}{\mathrm{NPHI}-1}. \tag{3.330}$$

The fifth statement in DO loop 105 sets

$$\mathrm{PHI1(J)} = \phi_J^{(1)}. \tag{3.331}$$

Equation (3.328) is written with the understanding that the term proportional to $V_{pq}^{1\mathrm{TM}}$ is to be omitted when $p = 0$ or $q = 0$. Similarly, substitution of $\phi_J^{(A2)}$ for ϕ in eq. (7.13) of [2] with $\gamma = 2$ yields

$$\frac{E_\phi^{(A2)}(\phi_J^{(A2)},0)}{|E_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}} = (S_a)\left(\frac{x_o}{a}\right)\sum_{q=0}^{\mathrm{NMAX}-1}\sum_{\substack{p=0\\p+q\neq 0}}^{\mathrm{MM}(q+1)-1}\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\left(\frac{1}{k_{pq}b}\right)$$
$$\cdot\left\{\left(\frac{pa}{b}\right)\left(\frac{V_{pq}^{2\mathrm{TM}}e^{j\beta_{01}^{\mathrm{TM}}L_3}}{\eta}\right)-\left(\frac{qa}{c}\right)\left(\frac{V_{pq}^{2\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}}{\eta}\right)\right\}$$
$$\cdot\sin\frac{q\pi}{2}\cos\left(p\phi_J^{(2)}\right) \tag{3.332}$$

[†]The ranges of values of p and q are taken to be the same as the respective ranges of values of m and n in (3.5).

where

$$\phi_{\mathrm{J}}^{(2)} = \frac{(\mathrm{J}-1)\pi}{\mathrm{NPHI}-1}. \tag{3.333}$$

The $V_{pq}^{2\mathrm{TM}}$ term should be omitted from (3.332) when $p=0$ or $q=0$. The sixth statement in DO loop 105 sets

$$\mathrm{PHI2(J)} = \phi_{\mathrm{J}}^{(2)}. \tag{3.334}$$

In DO loop 106, E3A1Z(J) and E3A2Z(J) are set to zero. Later,

$$\frac{E_z^{(A1)}(\pi, z_{\mathrm{J}}^{(A)})}{|E_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}} \text{ and } \frac{E_z^{(A2)}(0, z_{\mathrm{J}}^{(A)})}{|E_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}},$$

given by eq. (7.14) of [2] with $\gamma = 1$ and $\gamma = 2$, will be accumulated in E3A1Z(J) and E3A2Z(J), respectively. Here,

$$z_{\mathrm{J}}^{(A)} = \left(-1 + 2\frac{\mathrm{J}-1}{\mathrm{NZ}-1}\right)\frac{c}{2}. \tag{3.335}$$

Substituting $z_{\mathrm{J}}^{(A)}$ for z in eq. (7.14) of [2] with $\gamma = 1$ and placing the same upper limits on p and q as in (3.328), we obtain

$$\begin{aligned}\frac{E_z^{(A1)}(\pi, z_{\mathrm{J}}^{(A)})}{|E_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}} = S_a \sum_{q=0}^{\mathrm{NMAX}-1} \sum_{\substack{p=0\\p+q\neq 0}}^{\mathrm{MM}(q+1)-1} \sqrt{\frac{\epsilon_p\epsilon_q}{4}}\left(\frac{1}{k_{pq}b}\right)\\ \cdot\left\{\left(\frac{qa}{c}\right)\left(\frac{V_{pq}^{1\mathrm{TM}}e^{j\beta_{01}^{\mathrm{TM}}L_3}}{\eta}\right) + \left(\frac{pa}{b}\right)\left(\frac{V_{pq}^{1\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}}{\eta}\right)\right\}\\ \cdot \sin\frac{p\pi}{2}\cos\left(qz_{\mathrm{J}}^{(1)}\right)\end{aligned} \tag{3.336}$$

where

$$z_{\mathrm{J}}^{(1)} = \frac{(\mathrm{J}-1)\pi}{\mathrm{NZ}-1}. \tag{3.337}$$

The fourth statement in DO loop 106 sets

$$\mathrm{Z(J)} = z_{\mathrm{J}}^{(1)}. \tag{3.338}$$

The $V_{pq}^{1\mathrm{TM}}$ term should be omitted from (3.336) when $p = 0$ or $q = 0$. Similarly, substitution of $z_{\mathrm{J}}^{(A)}$ for z in eq. (7.14) of [2] with $\gamma = 2$ yields

$$\frac{E_z^{(A2)}(0, z_{\mathrm{J}}^{(A)})}{|E_{01}^{\mathrm{TMe}+}|_{\mathrm{rms}}} = S_a \sum_{q=0}^{\mathrm{NMAX}-1} \sum_{\substack{p=0 \\ p+q\neq 0}}^{\mathrm{MM}(q+1)-1} \sqrt{\frac{\epsilon_p \epsilon_q}{4}} \left(\frac{1}{k_{pq} b}\right)$$

$$\cdot \left\{ \left(\frac{qa}{c}\right) \left(\frac{V_{pq}^{2\mathrm{TM}} e^{j\beta_{01}^{\mathrm{TM}} L_3}}{\eta}\right) + \left(\frac{pa}{b}\right) \left(\frac{V_{pq}^{2\mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3}}{\eta}\right) \right\}$$

$$\cdot \sin \frac{p\pi}{2} \cos \left(q z_{\mathrm{J}}^{(1)}\right) . \tag{3.339}$$

The $V_{pq}^{2\mathrm{TM}}$ term should be omitted from (3.339) when $p = 0$ or $q = 0$.

Statement 155 and the statement after it set

$$\mathrm{ARG} = \beta_{01}^{\mathrm{TM}} L_3 \tag{3.340}$$

$$\mathrm{SA} = S_a \tag{3.341}$$

where S_a is given by (3.329). The second and third statements in DO loop 136 set

$$\mathrm{SINP(J)} = S_a \sin \frac{p\pi}{2} \tag{3.342}$$

$$\mathrm{SINQ(J)} = (S_a) \left(\frac{x_o}{a}\right) \sin \frac{q\pi}{2} \tag{3.343}$$

where

$$p = q = \mathrm{J} - 1. \tag{3.344}$$

Separate indices q and p were used in (3.342) and (3.343) because the right-hand side of (3.342) is intended to be a factor common to both (3.336) and (3.339) and the right-hand side of (3.343) is intended to be a factor common to both (3.328) and (3.332).

In nested DO loops 101 and 104, the qp term in each of (3.328), (3.332), (3.336), and (3.339) is taken into account for

$$q = \mathrm{Q} - 1 \tag{3.345}$$

$$p = \mathrm{P} - 1. \tag{3.346}$$

The third and seventh statements in DO loop 101 set

$$\mathrm{FQ1C} = \frac{qa}{c} \tag{3.347}$$

$$\mathrm{SINQQ} = (S_a)\left(\frac{x_o}{a}\right)\sin\frac{q\pi}{2}. \tag{3.348}$$

The subscripts JTM, JTM2, JTE1, and JTM2 are calculated inside DO loop 104 so that

$$\mathrm{V(JTM)} = \tfrac{1}{\eta}V_{pq}^{1\mathrm{TM}}e^{j\beta_{01}^{\mathrm{TM}}L_3} \tag{3.349}$$

$$\mathrm{V(JTM2)} = \tfrac{1}{\eta}V_{pq}^{2\mathrm{TM}}e^{j\beta_{01}^{\mathrm{TM}}L_3} \tag{3.350}$$

$$\mathrm{V(JTE1)} = \tfrac{1}{\eta}V_{pq}^{1\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3} \tag{3.351}$$

$$\mathrm{V(JTE2)} = \tfrac{1}{\eta}V_{pq}^{2\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}. \tag{3.352}$$

Before execution of the ninth statement in DO loop 104,

$$\mathrm{BMNJ} = \left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right). \tag{3.353}$$

The ninth through seventeenth statements in DO loop 104 set

$$\mathrm{SINPP} = S_a\sin\frac{p\pi}{2} \tag{3.354}$$

$$\mathrm{BMNJP} = \left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\left(\frac{pa}{b}\right) \tag{3.355}$$

$$\mathrm{BMNJQ} = \left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\left(\frac{qa}{c}\right) \tag{3.356}$$

$$\mathrm{BMNJPP} = (S_a)\left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\left(\frac{pa}{b}\right)\sin\frac{p\pi}{2} \tag{3.357}$$

$$\mathrm{BMNJQQ} = (S_a)\left(\frac{x_o}{a}\right)\left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\left(\frac{qa}{c}\right)\sin\frac{q\pi}{2} \tag{3.358}$$

$$\mathrm{YMM} = (S_a)\left(\frac{x_o}{a}\right)\left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\left(\frac{qa}{c}\right)$$

$$\cdot \left(\tfrac{1}{\eta} V_{pq}^{1\mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3}\right) \sin \frac{q\pi}{2} \tag{3.359}$$

$$\begin{aligned} \mathrm{YEM} &= (S_a)\left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\left(\frac{pa}{b}\right) \\ &\quad \cdot \left(\tfrac{1}{\eta} V_{pq}^{1\mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3}\right) \sin \frac{p\pi}{2} \end{aligned} \tag{3.360}$$

$$\begin{aligned} \mathrm{YME} &= -(S_a)\left(\frac{x_o}{a}\right)\left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\left(\frac{qa}{c}\right) \\ &\quad \cdot \left(\tfrac{1}{\eta} V_{pq}^{2\mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3}\right) \sin \frac{q\pi}{2} \end{aligned} \tag{3.361}$$

$$\begin{aligned} \mathrm{YEE} &= (S_a)\left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\left(\frac{pa}{b}\right) \\ &\quad \cdot \left(\tfrac{1}{\eta} V_{pq}^{2\mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3}\right) \sin \frac{p\pi}{2}. \end{aligned} \tag{3.362}$$

If the above values of YMM, YEM, YME, and YEE were multiplied by $\cos\left(p\phi_J^{(1)}\right)$, $\cos\left(qz_J^{(1)}\right)$, $\cos\left(p\phi_J^{(2)}\right)$, and $\cos\left(qz_J^{(1)}\right)$, respectively, then they would be the $V_{pq}^{1\mathrm{TE}}$ term in (3.328), the $V_{pq}^{1\mathrm{TE}}$ term in (3.336), $V_{pq}^{2\mathrm{TE}}$ term in (3.332), and the $V_{pq}^{2\mathrm{TE}}$ term in (3.339).

The eight statements before statement 134 account for the $V_{pq}^{1\mathrm{TM}}$ terms in (3.328) and (3.336) and the $V_{pq}^{2\mathrm{TM}}$ terms in (3.332) and (3.339). Since these terms are absent when $p = 0$ or $q = 0$, the eight statements before statement 134 are not executed when $p = 0$ or $q = 0$. The six statements before statement 134 set

$$\mathrm{BMNJPQ} = (S_a)\left(\frac{x_o}{a}\right)\left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\left(\frac{pa}{b}\right)\sin\frac{q\pi}{2} \tag{3.363}$$

$$\mathrm{BMNJQP} = (S_a)\left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\left(\frac{qa}{c}\right)\sin\frac{p\pi}{2} \tag{3.364}$$

$$\begin{aligned} \mathrm{YMM} &= -(S_a)\left(\frac{x_o}{a}\right)\left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\left\{\left(\frac{pa}{b}\right)\left(\tfrac{1}{\eta} V_{pq}^{1\mathrm{TM}} e^{j\beta_{01}^{\mathrm{TM}} L_3}\right)\right. \\ &\quad \left. -\left(\frac{qa}{c}\right)\left(\tfrac{1}{\eta} V_{pq}^{1\mathrm{TE}} e^{j\beta_{01}^{\mathrm{TM}} L_3}\right)\right\}\sin\frac{q\pi}{2} \end{aligned} \tag{3.365}$$

$$\mathrm{YEM} = (S_a)\left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\left\{\left(\frac{qa}{c}\right)\left(\tfrac{1}{\eta} V_{pq}^{1\mathrm{TM}} e^{j\beta_{01}^{\mathrm{TM}} L_3}\right)\right.$$

$$+\left(\frac{pa}{b}\right)\left(\frac{1}{\eta}V_{pq}^{1\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}\right)\Bigg\}\sin\frac{p\pi}{2} \tag{3.366}$$

$$\mathrm{YME} = (S_a)\left(\frac{x_o}{a}\right)\left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\Bigg\{\left(\frac{pa}{b}\right)\left(\frac{1}{\eta}V_{pq}^{2\mathrm{TM}}e^{j\beta_{01}^{\mathrm{TM}}L_3}\right)$$

$$-\left(\frac{qa}{c}\right)\left(\frac{1}{\eta}V_{pq}^{2\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}\right)\Bigg\}\sin\frac{q\pi}{2} \tag{3.367}$$

$$\mathrm{YEE} = (S_a)\left(\sqrt{\frac{\epsilon_p\epsilon_q}{4}}\right)\left(\frac{1}{k_{pq}b}\right)\Bigg\{\left(\frac{qa}{c}\right)\left(\frac{1}{\eta}V_{pq}^{2\mathrm{TM}}e^{j\beta_{01}^{\mathrm{TM}}L_3}\right)$$

$$+\left(\frac{pa}{b}\right)\left(\frac{1}{\eta}V_{pq}^{2\mathrm{TE}}e^{j\beta_{01}^{\mathrm{TM}}L_3}\right)\Bigg\}\sin\frac{p\pi}{2}. \tag{3.368}$$

If the above values of YMM, YEM, YME, and YEE were multiplied by $\cos\left(p\phi_{\mathrm{J}}^{(1)}\right)$, $\cos\left(qz_{\mathrm{J}}^{(1)}\right)$, $\cos\left(p\phi_{\mathrm{J}}^{(2)}\right)$, and $\cos\left(qz_{\mathrm{J}}^{(1)}\right)$, respectively, then they would be the *pq* terms† in (3.328), (3.336), (3.332) and (3.339), respectively.

The first statement in DO loop 135 adds the *pq* term in (3.328) to E3A1P(J). The second statement in DO loop 135 adds the *pq* term in (3.332) to E3A2P(J). The second statement in DO loop 148 adds the *pq* term in (3.336) to E3A1Z(J). The third statement in DO loop 148 adds the *pq* term in (3.339) to E3A2Z(J). Thus, upon exit from nested DO loops 101 and 104,

$$\mathrm{E3A1P(J)} = \frac{E_{\phi}^{(A1)}(\phi_{\mathrm{J}}^{(A1)},0)}{|E_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}} \tag{3.369}$$

$$\mathrm{E3A1Z(J)} = \frac{E_{z}^{(A1)}(\pi,z_{\mathrm{J}}^{(A)})}{|E_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}} \tag{3.370}$$

$$\mathrm{E3A2P(J)} = \frac{E_{\phi}^{(A2)}(\phi_{\mathrm{J}}^{(A2)},0)}{|E_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}} \tag{3.371}$$

$$\mathrm{E3A2Z(J)} = \frac{E_{z}^{(A2)}(0,z_{\mathrm{J}}^{(A)})}{|E_{01}^{\mathrm{TMe+}}|_{\mathrm{rms}}}. \tag{3.372}$$

†The *pq* term in any one of these equations is the right-hand side of the equation without the summation signs.

The first and second statements in DO loop 137 set

$$\text{E3A1PS(J)} = \left|\frac{E_\phi^{(A1)}(\phi_\text{J}^{(A1)},0)}{|E_{01}^{\text{TMe}+}|_{\text{rms}}}\right| \tag{3.373}$$

$$\text{E3A2PS(J)} = \left|\frac{E_\phi^{(A2)}(\phi_\text{J}^{(A2)},0)}{|E_{01}^{\text{TMe}+}|_{\text{rms}}}\right|. \tag{3.374}$$

The first and second statements in DO loop 138 set

$$\text{E3A1ZS(J)} = \left|\frac{E_z^{(A1)}(\pi,z_\text{J}^{(A)})}{|E_{01}^{\text{TMe}+}|_{\text{rms}}}\right| \tag{3.375}$$

$$\text{E3A2ZS(J)} = \left|\frac{E_z^{(A2)}(0,z_\text{J}^{(A)})}{|E_{01}^{\text{TMe}+}|_{\text{rms}}}\right|. \tag{3.376}$$

3.11 Listing of the Main Program

```
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /MODES/PC,BKM2,KTM,KTE,MM(50),BMN(100),BMN2(100)
      COMMON /PI/PI
      COMMON /NMAX/NMAX
      COMMON /BES/XM,SMAX
      COMMON /PHI/BX,BX5,PMAX,R,SGR,PH1(100),PH2(100),PH3(100),
     1PH4(100)
      COMMON /DGN/S,BKA2,L3,C,C5,PI5,D3(50),G4(50),PGC
      COMPLEX*16 ZL1,ZL2,U,BKU,SA,YTE,YTM,DTM(50),DTMN,DTE(50),DTEN
      COMPLEX*16 D3,D3N,Z1,Z2,Z3,Z4,Z5,S1,S3,S4,S5
      COMPLEX*8 E3A1P(100),E3A1Z(100),E3A2P(100),E3A2Z(100)
      COMPLEX*8 YMM,YEM,YME,YEE,Y(24336),TI(156),V(156),VI,CVTME(156)
      COMPLEX*8 CVTEE(156),CVTEO(156),CTME,CTEE,CTEO,C1OUT,C1IN,C2OUT
      COMPLEX*8 C2IN,YREC(156)
      REAL*8 L1,L2,L3,XJ(200),XJP(200),GTM(50),GTE(50)
      REAL*8 TMP(50),TMM(50),TEP(50),TEM(50),DQTM(50),DQTE(50)
      REAL*4 PHI1(100),PHI2(100),Z(100)
      REAL*4 PTRAN(100),PREFL(100),BKAPLT(100)
      REAL*4 C1OUTS,C1INS,C2OUTS,C2INS,PT,CTMES,CTEES,CTEOS,PR,PTOTAL
      REAL*4 SINP(100),SINQ(100),E3A1PS(100),E3A2PS(100),E3A1ZS(100)
      REAL*4 E3A2ZS(100),PTMS,PRECMS,PTMR,BKAG,SINQQ,SINPP,BMNJP
      REAL*4 BMNJQ,BMNJPP,BMNJQQ,BMNJPQ,BMNJQP
      INTEGER R,R1,S,SMAX,P,PMAX,P1,P2,P3,Q,Q1,PTM,PTE,QTM,QTE,IPS(156)
```

```
      INTEGER QPN,KE3(101)
      OPEN(UNIT=20,FILE='IN.DAT',STATUS='OLD')
      OPEN(UNIT=21,FILE='OUT.DAT',STATUS='OLD')
      READ(20,10) B,C,L1,L2,L3,BKM,XM,ZL1,ZL2
   10 FORMAT(4D14.7)
      WRITE(21,11)
   11 FORMAT('B C,L1,L2,L3,BKM,XM,ZL1,ZL2')
      WRITE(21,10) B,C,L1,L2,L3,BKM,XM,ZL1,ZL2
      READ(20,144) KAM,BKAO,DBKA,KE3M,NPHI,NZ
  144 FORMAT(I4,2D14.7,3I4)
      WRITE(21,145) KAM,BKAO,DBKA,KE3M,NPHI,NZ
  145 FORMAT('KAM=',I4,', BKAO=',D14.7,', DBKA=',D14.7/
     1'KE3M=',I4,', NPHI=',I4,', NZ=',I4)
      READ(20,146)(KE3(I),I=1,KE3M)
  146 FORMAT(15I4)
      WRITE(21,147)(KE3(I),I=1,KE3M)
  147 FORMAT('KE3'/(15I4))
      PI=3.14159265358979D+0
      BC=B/C
      PC=PI*BC
      BKM2=BKM*BKM
      CALL MODES
      WRITE(21,102)(MM(I),I=1,NMAX)
  102 FORMAT(10I4)
      K1=KTM+KTE
      WRITE(21,153) KTM,KTE,K1
  153 FORMAT('KTM=',I4,', KTE=',I4,', K1=',I4)
      IF(NMAX.GT.0) GO TO 115
      WRITE(21,116)
  116 FORMAT('BKM IS TOO SMALL')
      STOP
  115 CALL BESIN
      PI2=PI*2.D+0
      PIBC=PI*BC
      PI5=PI*.5D+0
      KAE=1
      DO 48 KA=1,KAM
      BKA=BKAO+(KA-1)*DBKA
      BKA2=BKA*BKA
      IF(BKA.GT.2.40482556D+0) GO TO 94
      WRITE(21,95)
   95 FORMAT('BKA IS TOO SMALL')
      STOP
   94 IF(BKA.LT.3.05423693D+0) GO TO 96
```

```
      WRITE(21,97)
   97 FORMAT('BKA IS TOO LARGE')
      STOP
   96 IF(C.LT.B) GO TO 98
      WRITE(21,99)
   99 FORMAT('C IS NOT LESS THAN B')
      STOP
   98 BKB=BKA*B
      WRITE(21,150) BKB
  150 FORMAT('BKB=',E14.7)
      IF(BKB.GT.PI) GO TO 110
      WRITE(21,111)
  111 FORMAT('BKB IS TOO SMALL')
      STOP
  110 IF(BKB.LT.PI2.AND.BKB.LT.PIBC) GO TO 112
      WRITE(21,113)
  113 FORMAT('BKB IS TOO LARGE')
      STOP
  112 BKB2=BKB*BKB
      BKR=1.D+0/BKB
      U=(0.D+0,1.D+0)
      BKU=-BKR*U
      B5=B*.5D+0
      BX5=DASIN(B5)
      BX=2.D+0*BX5
      XB=1.D+0/BX
      X1=L1/B-XB
      X2=L2/B-XB
      JTE=0
      JTM=0
      DO 13 Q=1,NMAX
      P2=1
      IF(Q.EQ.1) P2=2
      P3=MM(Q)
      DO 14 P=P2,P3
      JTE=JTE+1
      JTE1=JTE+KTM
      JTE2=JTE1+K1
      GAM2=BMN2(JTE)-BKB2
      IF(P.NE.2.OR.Q.NE.1) GO TO 15
      BET=DSQRT(-GAM2)
      A1=BET*X1
      CA=DCOS(A1)
      SA=DSIN(A1)*U
```

```
      S1=BET*BKU
      YREC(JTE1)=(CA+ZL1*SA)/(SA+ZL1*CA)*S1
      A2=BET*X2
      CA=DCOS(A2)
      SA=DSIN(A2)*U
      YREC(JTE2)=(CA+ZL2*SA)/(SA+ZL2*CA)*S1
      GO TO 17
   15 GAM=DSQRT(GAM2)
      YTE=-GAM*BKR
      YREC(JTE1)=YTE
      YREC(JTE2)=YTE
   17 IF(P.EQ.1.OR.Q.EQ.1) GO TO 14
      YTM=BKB/GAM
      JTM=JTM+1
      JTM2=JTM+K1
      YREC(JTM)=YTM
      YREC(JTM2)=YTM
   14 CONTINUE
   13 CONTINUE
      K2=K1*2
      WRITE(21,100)(YREC(J),J=1,K2)
  100 FORMAT('YREC'/(4E14.7))
      C5=C*.5D+0
      ZSS=1.D+0/C5
      PMAX=MM(1)
      XZ=B/BX
      TZTM=8.*BX5*DSQRT(BC/PI)
      TZTE=TZTM/BC
      TA=PI2/BKA
      SQ2=1.D+0/DSQRT(2.D+0)
      TC1=DSQRT(PI*BC)
      TC5=TC1/SQ2
      TC1=TC1*BKA
      SN1=B5
      CS1=DSQRT(1.D+0-B5*B5)
      K3=K2*K1
      DO 12 IY=1,K3
      Y(IY)=0.
   12 CONTINUE
      SGR=-1.D+0
      DO 19 R=1,500
      SGR=-SGR
      R1=R-1
      RS=R1*R1
```

```
      CALL BES(R,XJ,XJP)
      IF(SMAX.GT.200) STOP 60
      IF(SMAX.EQ.0) GO TO 25
      CALL PHI
      DO 20 S=1,SMAX
      CALL DGN(1,XJ,XXTM,ITM,GAMTM,TMP,TMM,DTM,GTM,DQTM,GCSTM,GC2TM,
     1ZEETM,ZZTM,ZOETM,ZOOTM)
      CALL DGN(2,XJP,XXTE,ITE,GAMTE,TEP,TEM,DTE,GTE,DQTE,GCSTE,GC2TE,
     1ZEETE,ZZTE,ZOETE,ZOOTE)
      XR=XXTE-RS
      W2=-RS/XR*GAMTE
      W6=XZ*XXTE
      W5=W6/XR
      W3=R1*W5
      W5=W5*W6
      W6=ZSS/XXTE*W5
      W1=BKA2/GAMTM
      W5=W5/GAMTE
      IF(R1.NE.0) GO TO 46
      W1=W1*.5D+0
      W5=W5*.5D+0
      W6=W6*.5D+0
   46 IF(R1.GT.1.OR.S.GT.1) GO TO 68
      IF(R1.EQ.1) GO TO 71
      MTM=0
      MTE=0
      DO 29 N=1,NMAX
      M2=1
      N1=N-1
      FN1=N1
      TITM=TZTM*FN1
      TITE=TZTE
      IF(N1.NE.0) GO TO 76
      M2=2
      TITE=TITE*SQ2
   76 M3=MM(N)
      DO 52 M=M2,M3
      M1=M-1
      IF(M1.NE.2*(M1/2)) GO TO 65
      IF(N1.EQ.0.OR.M1.EQ.0) GO TO 66
      MTM=MTM+1
      TI(MTM)=0.
      J=MTM+K1
      TI(J)=0.
```

```
 66 MTE=MTE+1
    J=MTE+KTM
    TI(J)=0.
    J=J+K1
    TI(J)=0.
    GO TO 52
 65 MTE=MTE+1
    F1=GTM(N)/BMN(MTE)
    IF(N1.EQ.0.OR.M1.EQ.0) GO TO 67
    MTM=MTM+1
    FM1=M1
    SA=-TITM/FM1*F1*U
    TI(MTM)=SA
    J=MTM+K1
    TI(J)=SA
 67 SA=-TITE*F1*U
    J=MTE+KTM
    TI(J)=SA
    J=J+K1
    TI(J)=SA
 52 CONTINUE
 29 CONTINUE
    TC2=TC1/GAMTM
    MTM=0
    MTE=0
    DO 77 N=1,NMAX
    M2=1
    N1=N-1
    M3=MM(N)
    TC3=TC2*GTM(N)
    IF(N1.NE.0) GO TO 114
    M2=2
    TC3=TC3*SQ2
114 DO 78 M=M2,M3
    MTE=MTE+1
    TC4=TC3/BMN(MTE)*PH2(M)
    M1=M-1
    IF(M1.EQ.0) TC4=TC4*SQ2
    IF(N1.EQ.0.OR.M1.EQ.0) GO TO 79
    MTM=MTM+1
    TC4N=N1*TC4
    CVTME(MTM)=TC4N
    J=MTM+K1
    CVTME(J)=TC4N
```

```
79 TC4M=M1/BC*TC4
   J=MTE+KTM
   CVTME(J)=TC4M
   J=J+K1
   CVTME(J)=TC4M
78 CONTINUE
77 CONTINUE
   GAMO1=GAMTM
   GO TO 68
71 TC2=TC5*DSQRT(GAMTE/(GAMO1*XR))
   TC6=XZ*XXTE/GAMTE
   MTM=0
   MTE=0
   DO 80 N=1,NMAX
   M2=1
   N1=N-1
   TC3=TC2
   IF(N1.NE.0) GO TO 16
   M2=2
   TC3=TC3*SQ2
16 M3=MM(N)
   TC7=TC3*TC6*G4(N)
   TC3=TC3*GTE(N)
   FN1=N1
   DO 73 M=M2,M3
   M1=M-1
75 MTE=MTE+1
   BMNM=BMN(MTE)
   TC4=TC3/BMNM
   TC8=TC7/BMNM
   IF(M1.NE.0) GO TO 74
   TC4=TC4*SQ2
   TC8=TC8*SQ2
74 PH1M=PH1(M)
   PH2M=PH2(M)
   PH3M=PH3(M)
   PH4M=PH4(M)
   PAG1=TC4*(PH1M*CS1-PH2M*SN1)
   PAG2=TC4*(PH2M*CS1+PH1M*SN1)
   PAG3=TC8*(PH3M*CS1-PH4M*SN1)
   PAG4=TC8*(PH4M*CS1+PH3M*SN1)
   FM1B=M1/BC
   IF(N1.EQ.0.OR.M1.EQ.0) GO TO 72
   MTM=MTM+1
```

```
      PAG5=PAG1*FN1+PAG4*FM1B
      PAG6=PAG2*FN1-PAG3*FM1B
      CVTEE(MTM)=PAG5
      CVTEO(MTM)=PAG6
      J=MTM+K1
      CVTEE(J)=PAG5
      CVTEO(J)=-PAG6
   72 PAG5=PAG1*FM1B-PAG4*FN1
      PAG6=PAG2*FM1B+PAG3*FN1
      J=MTE+KTM
      CVTEE(J)=PAG5
      CVTEO(J)=PAG6
      J=J+K1
      CVTEE(J)=PAG5
      CVTEO(J)=-PAG6
   73 CONTINUE
   80 CONTINUE
   68 NTM=0
      NTE=0
      DO 21 N=1,NMAX
      M2=1
      N1=N-1
      IF(N1.EQ.0) M2=2
      M3=MM(N)
      QTM=0
      QTE=0
      FN1B=N1*BC
      NEO=1+N1-2*(N1/2)
      GO TO (18,51), ITM
   18 TMMN=TMM(N)
      TMPN=TMP(N)
      DTMN=DTM(N)
      GO TO 83
   51 DQTMN=DQTM(N)
      TM1=GCSTM*DQTMN
      TM2=GC2TM*DQTMN
   83 GO TO (84,85), ITE
   84 TEMN=TEM(N)
      TEPN=TEP(N)
      DTEN=DTE(N)
      D3N=D3(N)
      GO TO 86
   85 DQTEN=DQTE(N)
      TE1=GCSTE*DQTEN
```

```
      TE2=GC2TE*DQTEN
      FN1=N1
      PNG=FN1*PGC
   86 DO 22 Q=1,NMAX
      P2=1
      Q1=Q-1
      IF(Q1.EQ.0) P2=2
      P3=MM(Q)
      W8=Q1*FN1B
      NQEO=NEO+Q1-2*(Q1/2)
      NEQQ=2
      IF(N1.EQ.Q1.AND.Q1.NE.0) NEQQ=1
      QPN=2
      IF(Q1.EQ.0.AND.N1.EQ.0) QPN=1
      GO TO (31,32), ITM
   31 IM=N1-Q1
      IP=N1+Q1
      TMMQ=TMM(Q)
      TMPQ=TMP(Q)
      FTM=FXY(IP,TMPN,TMMQ)-FXY(IM,TMMN,-TMMQ)
     1-FXY(IP,TMMN,TMPQ)+FXY(IM,TMPN,-TMPQ)
      Z1=W1*(FTM+GTM(Q)*DTMN)
      GO TO 47
   32 GO TO (45,107,117), NQEO
   45 GO TO (118,119), QPN
  118 Z1R=ZZTM
      GO TO 120
  119 Z1R=ZEETM
      GO TO 120
  107 Z1R=ZOETM
      GO TO 120
  117 Z1R=ZOOTM
  120 Z1R=Z1R*TM1*DQTM(Q)
      GO TO (121,122), NEQQ
  121 Z1R=Z1R-TM2
  122 Z1R=-W1*Z1R
   47 GO TO (36,37) ITE
   36 GTEQ=GTE(Q)
      G4Q=G4(Q)
      IM=N1-Q1
      IP=N1+Q1
      TEMQ=TEM(Q)
      TEPQ=TEP(Q)
      F1=FXY(IM,TEMN,-TEMQ)
```

```
      F2=FXY(IP,TEPN,TEMQ)
      F6=FXY(IP,TEMN,TEPQ)
      F7=FXY(IM,TEPN,-TEPQ)
      F8=F2-F1
      F9=F2+F1
      F1=F6-F7
      F2=F6+F7
      FTE=F8-F1
      F3=F9+F2
      F4=F8+F1
      F5=F9-F2
      Z2=W2*(FTE+GTEQ*DTEN)
      Z3=W3*(F3+GTEQ*D3N)
      Z4=-W3*(F4+G4Q*DTEN)
      Z5=W5*(F5+G4Q*D3N)
      IF(N1.EQ.Q1.AND.N1.NE.0) Z5=W6+Z5
      GO TO 49
   37 GO TO (123,124,125), NQEO
  123 GO TO (126,127), QPN
  126 ZR=ZZTE
      GO TO 128
  127 ZR=ZEETE
      GO TO 128
  124 ZR=ZOETE
      GO TO 128
  125 ZR=ZOOTE
  128 ZR=ZR*TE1*DQTE(Q)
      Z2R=ZR
      GO TO (129,130), NEQQ
  129 Z2R=Z2R-TE2
  130 Z3R=PNG*Z2R
      FQ1=Q1
      PQG=FQ1*PGC
      Z4R=PQG*Z2R
      Z5R=PNG*PQG*ZR
      GO TO (131,132), NEQQ
  131 Z5R=Z5R+TE2
  132 Z2R=W2*Z2R
      Z3R=W3*Z3R
      Z4R=W3*Z4R
      Z5R=-W5*Z5R
      GO TO (133,49) NEQQ
  133 Z5R=W6+Z5R
   49 IF((ITM+ITE).NE.3) GO TO 87
```

```
      GO TO (53,54),ITM
   53 Z2=Z2R
      Z3=Z3R
      Z4=Z4R
      Z5=Z5R
      GO TO 87
   54 Z1=Z1R
   87 MTM=NTM
      MTE=NTE
      DO 23 M=M2,M3
      KMN=1
      M1=M-1
      IF(N1.EQ.0.OR.M1.EQ.0) GO TO 26
      KMN=2
      MTM=MTM+1
   26 MTE=MTE+1
      W9=M1*Q1
      FM1B=M1/BC
      TB=TA/BMN(MTE)
      PTM=QTM
      PTE=QTE
      DO 24 P=P2,P3
      KPQ=1
      P1=P-1
      IF(Q1.EQ.0.OR.P1.EQ.0) GO TO 27
      KPQ=2
      PTM=PTM+1
   27 PTE=PTE+1
      W10=N1*P1
      W11=P1*FM1B
      T1=TB/BMN(PTE)
      IF(M1.EQ.0) T1=T1*SQ2
      IF(N1.EQ.0) T1=T1*SQ2
      IF(P1.EQ.0) T1=T1*SQ2
      IF(Q1.EQ.0) T1=T1*SQ2
      PH1P=PH1(P)
      PH2P=PH2(P)
      PH3P=PH3(P)
      PH4P=PH4(P)
      KB=1+(KMN+KPQ)/4
      KM=(PTM-1)*K2
      KE1=KTM+MTE
      KMM=KM+MTM
      KEM=KM+KE1
```

```
      KE2=(KTM+PTE-1)*K2
      KME=KE2+MTM
      KEE=KE2+KE1
      K=0
      MJ=M
      DO 28 J=1,2
      PH1MJ=PH1(MJ)
      PH2MJ=PH2(MJ)
      PH3MJ=PH3(MJ)
      PH4MJ=PH4(MJ)
      PAG1=PH2P*PH3MJ-PH1P*PH4MJ
      PAG2=PH2P*PH2MJ+PH1P*PH1MJ
      PAG3=PH4P*PH1MJ-PH3P*PH2MJ
      PAG4=PH4P*PH4MJ+PH3P*PH3MJ
      IF(J.EQ.1) GO TO 30
      PAG1=-PAG1
      PAG4=-PAG4
   30 IF((ITE+ITM).EQ.4) GO TO 56
   50 S1=PAG2*(Z1-Z2)
      S3=PAG1*Z3
      S4=PAG3*Z4
      S5=PAG4*Z5
   64 GO TO (33,38), KB
   38 IMM=KMM+K
      YMM=T1*(W8*S1+W9*S3-W10*S4-W11*S5)
      Y(IMM)=Y(IMM)+YMM
   33 GO TO (34,39),KPQ
   39 IEM=KEM+K
      YEM=T1*(W9*S1-W8*S3-W11*S4+W10*S5)
      Y(IEM)=Y(IEM)+YEM
   34 GO TO (35,40),KMM
   40 IME=KME+K
      YME=T1*(W10*S1+W11*S3+W8*S4+W9*S5)
      Y(IME)=Y(IME)+YME
   35 IEE=KEE+K
      YEE=T1*(W11*S1-W10*S3+W9*S4-W8*S5)
      Y(IEE)=Y(IEE)+YEE
      GO TO 57
   56 S1R=PAG2*(Z1R-Z2R)
      S3R=PAG1*Z3R
      S4R=PAG3*Z4R
      S5R=PAG4*Z5R
      GO TO (58,59), KB
   59 IMM=KMM+K
```

```
      YMM=T1*(W8*S1R+W9*S3R-W10*S4R-W11*S5R)
      Y(IMM)=Y(IMM)+YMM
   58 GO TO (60,61), KPQ
   61 IEM=KEM+K
      YEM=T1*(W9*S1R-W8*S3R-W11*S4R+W10*S5R)
      Y(IEM)=Y(IEM)+YEM
   60 GO TO (62,63), KMN
   63 IME=KME+K
      YME=T1*(W10*S1R+W11*S3R+W8*S4R+W9*S5R)
      Y(IME)=Y(IME)+YME
   62 IEE=KEE+K
      YEE=T1*(W11*S1R-W10*S3R+W9*S4R-W8*S5R)
      Y(IEE)=Y(IEE)+YEE
   57 K=K1
      MJ=M+PMAX
   28 CONTINUE
   24 CONTINUE
   23 CONTINUE
      QTM=PTM
      QTE=PTE
   22 CONTINUE
      NTM=MTM
      NTE=MTE
   21 CONTINUE
   20 CONTINUE
   19 CONTINUE
      WRITE(21,44)
   44 FORMAT('R IS TOO LARGE')
      STOP
   25 IF(R.NE.1) GO TO 41
      WRITE(21,42)
   42 FORMAT ('XM IS TOO SMALL')
      STOP
   41 K4=K1*K2
      J1=0
      DO 89 J=1,K1
      DO 88 I=1,K1
      J1=J1+1
      J2=J1+K1
      J3=J1+K4
      J4=J3+K1
      Y(J3)=Y(J2)
      Y(J4)=Y(J1)
   88 CONTINUE
```

```
      J1=J1+K1
   89 CONTINUE
      K5=K2+1
      IY=1
      DO 43 I=1,K2
      Y(IY)=Y(IY)+YREC(I)
      IY=IY+K5
   43 CONTINUE
      WRITE(21,204)(TI(I),I=1,K2)
  204 FORMAT('TI'/(4E14.7))
      CALL DECOMP(K2,IPS,Y)
      CALL SOLVE(K2,IPS,Y,TI,V)
      WRITE(21,206)(V(I),I=1,K2)
  206 FORMAT('V'/(4E14.7))
      BET=DSQRT(BKB2-BMN2(1))/B
      ARG=BET*(L1-XZ)
      CS=DCOS(ARG)
      SN=DSIN(ARG)
      KV=KTM+1
      S1=.5D+0*DSQRT(BET/GAM01)
      SA=S1*V(KV)/(CS*ZL1+U*SN)
      C1OUT=(ZL1+1.D+0)*DCMPLX(CS,SN)*SA
      C1IN=(ZL1-1.D+0)*DCMPLX(CS,-SN)*SA
      WRITE(21,69) C1OUT,C1IN
   69 FORMAT('C1OUT=',2E14.7,', C1IN=',2E14.7)
      ARG=BET*(L2-XZ)
      CS=DCOS(ARG)
      SN=DSIN(ARG)
      KV=KV+K1
      SA=S1*V(KV)/(CS*ZL2+U*SN)
      C2OUT=(ZL2+1.D+0)*DCMPLX(CS,SN)*SA
      C2IN=(ZL2-1.D+0)*DCMPLX(CS,-SN)*SA
      WRITE(21,70) C2OUT,C2IN
   70 FORMAT('C2OUT=',2E14.7,', C2IN=',2E14.7)
      CTME=0.
      CTEE=0.
      CTEO=0.
      DO 55 I=1,K2
      VI=V(I)
      CTME=CTME+CVTME(I)*VI
      CTEE=CTEE+CVTEE(I)*VI
      CTEO=CTEO+CVTEO(I)*VI
   55 CONTINUE
      CTMMS=CTME*CONJG(CTME)
```

```
      CTME=CTME-1.D+0
      WRITE(21,81) CTME,CTEE,CTEO
   81 FORMAT('CTME=',2E14.7,', CTEE=',2E14.7/'CTEO=',2E14.7)
      C1OUTS=C1OUT*CONJG(C1OUT)
      C1INS=C1IN*CONJG(C1IN)
      C2OUTS=C2OUT*CONJG(C2OUT)
      C2INS=C2IN*CONJG(C2IN)
      PT=C1OUTS-C1INS+C2OUTS-C2INS
      WRITE(21,82) C1OUTS,C1INS,C2OUTS,C2INS,PT
   82 FORMAT('C1OUTS=',E14.7,', C1INS=',E14.7,', C2OUTS=',E14.7/
     1'C2INS=',E14.7,', PT=',E14.7)
      CTMES=CTME*CONJG(CTME)
      CTEES=CTEE*CONJG(CTEE)
      CTEOS=CTEO*CONJG(CTEO)
      PR=CTMES+CTEES+CTEOS
      PRM=CTMMS+CTEES+CTEOS
      WRITE(21,90) CTMES,CTMMS,CTEES,CTEOS,PR,PRM
   90 FORMAT('CTMES=',E14.7,' ,CTMMS=',E14.7/'CTEES=',E14.7,
     1' CTEOS=',E14.7/'PR=',E14.7,' ,PRM=',E14.7)
      BKAPLT(KA)=BKA
      PTRAN(KA)=PT
      PREFL(KA)=PR
      PTOTAL=PT+PR
      WRITE(21,154) PTOTAL
  154 FORMAT('PTOTAL=',E14.7)
      YMM=0
      YME=0
      DO 91 I=1,K2
      YEE=CONJG(V(I))
      YMM=YMM+YREC(I)*V(I)*YEE
      YME=YME+TI(I)*YEE
   91 CONTINUE
      BKAG=BKA/GAM01
      PTA=-BKAG*AIMAG(YMM)
      PRMA=-PTA-BKAG*AIMAG(YME)
      WRITE(21,93) PTA,PRMA
   93 FORMAT('PTA=',E14.7,', PRMA=',E14.7)
      IF(KA.NE.KE3(KAE)) GO TO 48
      KAE=KAE+1
      DEL=PI/(NPHI-1)
      DO 105 J=1,NPHI
      E3A1P(J)=0.
      E3A2P(J)=0.
      FJ1=J-1
```

```
      FJ1D=FJ1*DEL
      PHI1(J)=PI-FJ1D
      PHI2(J)=FJ1D
  105 CONTINUE
      DEL=PI/(NZ-1)
      DO 106 J=1,NZ
      E3A1Z(J)=0.
      E3A2Z(J)=0.
      FJ1=J-1
      Z(J)=FJ1*DEL
  106 CONTINUE
      IF(KAE.NE.2) GO TO 155
      WRITE(21,152)(PHI2(I),I=1,NPHI)
  152 FORMAT('PHI2'/(5E14.7))
      WRITE(21,151)(Z(I),I=1,NZ)
  151 FORMAT('Z'/(5E14.7))
  155 ARG=GAMO1*L3
      SA=PI2*BKAG*DSQRT(PI*BC)*(DCOS(ARG)-U*DSIN(ARG))
      DO 136 J=1,PMAX
      FJ1=J-1
      SINP(J)=DSIN(FJ1*PI5)*SA
      SINQ(J)=SINP(J)*XZ
  136 CONTINUE
      JTM=0
      JTE=0
      DO 101 Q=1,NMAX
      Q1=Q-1
      FQ1=Q1
      FQ1C=FQ1/C
      P2=1
      IF(Q1.EQ.0) P2=2
      P3=MM(Q)
      SINQQ=SINQ(Q)
      DO 104 P=P2,P3
      JTE=JTE+1
      JTE1=JTE+KTM
      JTE2=JTE1+K1
      BMNJ=1./BMN(JTE)
      P1=P-1
      FP1=P1
      IF(Q1.EQ.0) BMNJ=BMNJ*SQ2
      IF(P1.EQ.0) BMNJ=BMNJ*SQ2
      SINPP=SINP(P)
      BMNJP=BMNJ*FP1/B
```

```
      BMNJQ=BMNJ*FQ1C
      BMNJPP=BMNJP*SINPP
      BMNJQQ=BMNJQ*SINQQ
      YMM=BMNJQQ*V(JTE1)
      YEM=BMNJPP*V(JTE1)
      YME=-BMNJQQ*V(JTE2)
      YEE=BMNJPP*V(JTE2)
      IF(P1.EQ.0.OR.Q1.EQ.0) GO TO 134
      JTM=JTM+1
      JTM2=JTM+K1
      BMNJPQ=BMNJP*SINQQ
      BMNJQP=BMNJQ*SINPP
      YMM=-BMNJPQ*V(JTM)+YMM
      YEM=BMNJQP*V(JTM)+YEM
      YME=BMNJPQ*V(JTM2)+YME
      YEE=BMNJQP*V(JTM2)+YEE
  134 PT=FP1
      DO 135 J=1,NPHI
      E3A1P(J)=E3A1P(J)+YMM*COS(PT*PHI1(J))
      E3A2P(J)=E3A2P(J)+YME*COS(PT*PHI2(J))
  135 CONTINUE
      PT=FQ1
      DO 148 J=1,NZ
      PR=COS(PT*Z(J))
      E3A1Z(J)=E3A1Z(J)+YEM*PR
      E3A2Z(J)=E3A2Z(J)+YEE*PR
  148 CONTINUE
  104 CONTINUE
  101 CONTINUE
      DO 137 J=1,NPHI
      E3A1PS(J)=CABS(E3A1P(J))
      E3A2PS(J)=CABS(E3A2P(J))
  137 CONTINUE
      DO 138 J=1,NZ
      E3A1ZS(J)=CABS(E3A1Z(J))
      E3A2ZS(J)=CABS(E3A2Z(J))
  138 CONTINUE
      WRITE(21,140)(E3A1PS(J),J=1,NPHI)
  140 FORMAT('E3A1PS'/(5E14.7))
      WRITE(21,141)(E3A1ZS(J),J=1,NZ)
  141 FORMAT('E3A1ZS'/(5E14.7))
      WRITE(21,142)(E3A2PS(J),J=1,NPHI)
  142 FORMAT('E3A2PS'/(5E14.7))
      WRITE(21,143)(E3A2ZS(J),J=1,NZ)
```

```
143 FORMAT('E3A2ZS'/(5E14.7))
 48 CONTINUE
    WRITE(21,149)(BKAPLT(I),I=1,KAM)
149 FORMAT('BKAPLT'/(5E14.7))
    WRITE(21,92)(PTRAN(I),I=1,KAM)
 92 FORMAT('PTRAN'/(5E14.7))
    WRITE(21.139)(PREFL(I),I=1,KAM)
139 FORMAT('PREFL'/(5E14.7))
    STOP
    END
```

Chapter 4

The Subroutine MODES

The subroutine MODES calculates all the values of $k_{mn}b$ such that $(k_{mn}b)^2$ does not exceed the value of the input variable BKM2. Here, k_{mn} is the cutoff wavenumber of the mn^{th} TM or TE rectangular waveguide mode. According to (2.6),

$$k_{mn}b = \sqrt{(m\pi)^2 + \left(\frac{n\pi b}{c}\right)^2} \,. \tag{4.1}$$

An expansion function is associated with each rectangular waveguide mode (see Appendix A of [2]).

4.1 Description of the Subroutine MODES

The input and output variables of the subroutine MODES are listed in the three common blocks labeled MODES, PI, and NMAX:†

```
COMMON /MODES/PC,BKM2,KTM,KTE,MM(50),BMN(100),BMN2(100)
COMMON /PI/PI
COMMON /NMAX/NMAX
```

Here, BKM2 is the input variable mentioned in the first sentence of this chapter, and PC and PI are input variables defined by

$$\text{PC} = \pi b/c \tag{4.2}$$

$$\text{PI} = \pi. \tag{4.3}$$

†See the listing of the subroutine MODES in Section 4.2.

The remaining variables KTM, KTE, MM, BMN, BMN2, and NMAX are output variables.

Nested DO loops 12 and 13 set

$$\mathrm{BMN2(KTE)} = (k_{mn}b)^2, \begin{cases} m = \mathrm{M}-1, & \mathrm{M} = \mathrm{M2}, \mathrm{M2}+1, \cdots, \mathrm{MM(N)} \\ n = \mathrm{N}-1, & \mathrm{N} = 1, 2, \cdots, \mathrm{NMAX} \end{cases} \tag{4.4}$$

where

$$\mathrm{KTE} = \mathrm{M} - 1 + \begin{cases} 0, & \mathrm{N} = 1 \\ \sum_{l=1}^{\mathrm{N}-1} \mathrm{MM}(l), & \mathrm{N} > 1 \end{cases} \tag{4.5}$$

$$\mathrm{M2} = \begin{cases} 2, & \mathrm{N} = 1 \\ 1, & \mathrm{N} = 2, 3, \cdots. \end{cases} \tag{4.6}$$

Moreover, MM(N) is such that

$$\mathrm{BMN2(KTE)} \leq \mathrm{BKM2} \ \text{ when } \ \mathrm{M} = \mathrm{MM(N)} \tag{4.7}$$

but

$$\mathrm{BMN2(KTE)} > \mathrm{BKM2} \ \text{ when } \ \mathrm{M} = \mathrm{MM(N)} + 1. \tag{4.8}$$

Furthermore, NMAX is such that

$$\mathrm{BMN2(KTE)} \leq \mathrm{BKM2} \ \text{ when } \ \mathrm{M} = \mathrm{M2} \ \text{ and } \ \mathrm{N} = \mathrm{NMAX} \tag{4.9}$$

but

$$\mathrm{BMN2(KTE)} > \mathrm{BKM2} \ \text{ when } \ \mathrm{M} = \mathrm{M2} \ \text{ and } \ \mathrm{N} = \mathrm{NMAX} + 1. \tag{4.10}$$

It is assumed that $\mathrm{MM(N)} \leq 100$ for $\mathrm{N} = 1, 2, \cdots, \mathrm{NMAX}$. If not, the statement "STOP 13" terminates execution. It is also assumed that $\mathrm{NMAX} \leq 100$. If not, the statement "STOP 12" terminates execution.

Nested DO loops 12 and 13 also set

$$\mathrm{BMN(KTE)} = k_{mn}b \tag{4.11}$$

where KTE, m, and n are the same as in (4.4). Upon exit from nested DO loops 12 and 13, KTE will be its value when

$$\left. \begin{array}{l} \mathrm{N} = \mathrm{NMAX} \\ \mathrm{M} = \mathrm{MM(NMAX)} \end{array} \right\}. \tag{4.12}$$

Substitution of (4.12) into (4.5) gives

$$\mathrm{KTE} = -1 + \sum_{l=1}^{\mathrm{NMAX}} \mathrm{MM}(l) \ . \tag{4.13}$$

Upon exit from nested DO loops 12 and 13, KTM will be KTE of (4.13) minus the number of times that either M = 1 or N = 1 in inner DO loop 13:

$$\mathrm{KTM} = \sum_{l=2}^{\mathrm{NMAX}} (\mathrm{MM}(l) - 1) \ . \tag{4.14}$$

4.2 Listing of the Subroutine MODES

```
      SUBROUTINE MODES
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /MODES/PC,BKM2,KTM,KTE,MM(50),BMN(100),BMN2(100)
      COMMON /PI/PI
      COMMON /NMAX/NMAX
      KTM=0
      KTE=0
      DO 12 N=1,101
      CC=PC*(N-1)
      C2=CC*CC
      M2=1
      IF(N.EQ.1) M2=2
      DO 13 M=M2,101
      BB=PI*(M-1)
      B2=C2+BB*BB
      IF(B2.GT.BKM2) GO TO 15
      KTE=KTE+1
      BMN2(KTE)=B2
      BMN(KTE)=DSQRT(B2)
      IF(M.EQ.1.OR.N.EQ.1) GO TO 13
      KTM=KTM+1
   13 CONTINUE
      STOP 13
   15 IF(M.EQ.M2) GO TO 14
      MM(N)=M-1
   12 CONTINUE
      STOP 12
   14 NMAX=N-1
      RETURN
      END
```

Chapter 5

The Subroutine BESIN

The subroutine BESIN puts data in the common block labeled BESIN†. These data will be used by the subroutine BES (see Chapter 6) to calculate roots of Bessel functions and their derivatives (see Appendix B of [2]).

5.1 Description of the Subroutine BESIN

In the common block labeled PI, PI is an input variable defined by

$$\mathrm{PI} = \pi. \tag{5.1}$$

The subroutine BESIN reads input data from the file BESIN.DAT, writes output data in the file BESOUT.DAT, and puts output data in the common block labeled BESIN. The data mentioned in the previous sentence are described below.

5.1.1 Tabulated Roots of Bessel Functions

The array AA that is read in and written out prior to execution of DO loop 12 contains alphameric data indicating that the array X in DO loop 12 contains roots of Bessel functions. In DO loop 12,

$$X(\mathrm{N},\mathrm{S}) = j_{\mathrm{N}-1,\mathrm{S}} \tag{5.2}$$

†This common block appears in the listing of the subroutine BESIN (see Section 5.2).

where the j's are roots of Bessel functions (see Appendix B of [2]) taken from Table I of [6].

The write statement in DO loop 11 and the write statement following DO loop 11 are inactivated because of the "C" in column 1. Removal of the "C" from both of these write statements would cause the roots (5.2) of Bessel functions to be written as they appear in Table I of [6] where the first 50 roots of a Bessel function of a given order appear as a column of 50 numbers.

5.1.2 Tabulated Roots of Derivatives of Bessel Functions

The array AA that is read in and written out prior to execution of DO loop 14 contains alphameric data indicating that the array XP in DO loop 14 contains roots of derivatives of Bessel functions. In DO loop 14,

$$\mathrm{XP(N,S)} = j'_{\mathrm{N-1,S}} \tag{5.3}$$

where the j''s are roots of derivatives of Bessel functions (see Appendix B of [2]) taken from Table II of [6].

The write statement in DO loop 24 and the write statement following DO loop 24 are inactivated because of the "C" in column 1. Removal of the "C" from both of these write statements would cause the roots (5.3) of derivatives of Bessel functions to be written as they appear in Table II of [6] where the first 50 roots of the derivative of a Bessel function of a given order appear as a column of 50 numbers.

5.1.3 Interpolation Data for Roots of Bessel Functions of Large Order

In this section, interpolation data are obtained for the parameters z, p_1, and p_2 that appear in eq. (B.5) of [2]. In the subroutine BES,† these data will be substituted into the right-hand side of eq. (B.12) of [2].

The data f, δ_m^2, and γ^4 tabulated in [6] must be used with care in eq. (B.12) of [2]. The additional subscript 0 or 1 on f, δ_m^2, and γ^4 in eq. (B.12) of [2] denotes evaluation at either the nearest smaller or the nearest larger

†See Chapter 6.

tabulated value of the argument. Any tabulated value of f, δ_m^2, or γ^4 that has no sign and that follows a number with a negative sign is assummed to be negative. If all tabulated values of f have l digits to the right of the decimal point, then δ_m^2 and γ^4 are given in units of 10^{-l}. In other words, the tabulated values of δ_m^2 and γ^4 must be multiplied by 10^{-l} before insertion into eq. (B.12) of [2]. Alternatively, one can multiply the tabulated values of f by 10^l, take the tabulated values of δ_m^2 and γ^4 as they stand, and then multiply the computed right-hand side of eq. (B.12) of [2] by 10^{-l}. The latter course of action will be taken for interpolation in the subroutine BES. Otherwise stated, the interpolation formula that will be used in the subroutine BES is

$$10^l f_p = (1-p)\left(10^l f_0\right) + p\left(10^l f_1\right) + E_2\delta_{m0}^2 + F_2\delta_{m1}^2 + M_4\gamma_0^4 + N_4\gamma_1^4 \quad (5.4)$$

rather than eq. (B.12) of [2] as it stands.

The alphameric data AA that are read in and written out immediately before Z is read in introduce Z. Here,

$$Z(\mathrm{I}) = 10^9 z(-\zeta) \text{ for } -\zeta = 0.1*(\mathrm{I}-1),\ \mathrm{I}=1,2,\cdots,76 \quad (5.5)$$

$$Z(\mathrm{I}) = 10^9\left(z\left(\frac{1}{\xi^2}\right) - \frac{2}{3}\xi^{-3}\right) \text{ for } \xi = 0.02*(\mathrm{I}-77),$$
$$\mathrm{I} = 77,78,\cdots,96 \quad (5.6)$$

where $z(-\zeta)$ is the parameter z that appears in eq. (B.5) of [2]. The argument $-\zeta$ in $z(-\zeta)$ is given by eq. (B.6) of [2]. The alphameric data AA that are read in and written out immediately before ZD2 is read in introduce ZD2. Here,

$$\mathrm{ZD2(I)} = \delta_m^2,\ \mathrm{I} = 1,2,\cdots,96 \quad (5.7)$$

where $\{\delta_m^2, \mathrm{I} = 1,2,\cdots,76\}$ are the modified second differences in the interpolation formula (5.4) for $10^9 z(-\zeta)$ of (5.5) and $\{\delta_m^2, \mathrm{I} = 77,78,\cdots,96\}$ are the modified second differences in the interpolation formula (5.4) for $10^9\left(z\left(\frac{1}{\xi^2}\right) - \frac{2}{3}\xi^{-3}\right)$ of (5.6) where

$$\xi = \frac{1}{\sqrt{-\zeta}}. \quad (5.8)$$

Formula (5.4) for $10^9 z(-\zeta)$ is (5.4) with f replaced by z. Formula (5.4) for $10^9 z(-\zeta)$ interpolates $10^9 z$ as a function of $-\zeta$ for $0 \le -\zeta < 7.5$. Formula

(5.4) for $10^9\left(z\left(\frac{1}{\xi^2}\right)-\frac{2}{3}\xi^{-3}\right)$ is (5.4) with f replaced by $z-\frac{2}{3}\xi^{-3}$. Formula (5.4) for $10^9\left(z\left(\frac{1}{\xi^2}\right)-\frac{2}{3}\xi^{-3}\right)$ interpolates $10^9\left(z-\frac{2}{3}\xi^{-3}\right)$ as a function of ξ for $\{0\leq\xi<0.38\}$. However, this formula will be used only for $0\leq\xi\leq\frac{1}{\sqrt{7.5}}$ because the range $\frac{1}{\sqrt{7.5}}<\xi<0.38$ lies within the range $0\leq-\zeta<7.5$. In (5.7), I denotes evaluation at the value of $-\zeta$ in (5.5) when $1\leq \mathrm{I}\leq 76$ and at the value of ξ in (5.6) when $77\leq \mathrm{I}\leq 96$. The alphameric data AA that are read in and written out immediately before ZD4 is read in introduce ZD4. Here,

$$\mathrm{ZD4(I)}=\gamma^4,\ \ \mathrm{I}=1,2,\cdots,96 \tag{5.9}$$

where $\{\gamma^4,\ \mathrm{I}=1,2,\cdots,96\}$ are the modified fourth differences that complement the modified second differences $\{\delta_m^2,\ \mathrm{I}=1,2,\cdots,96\}$ in (5.7).

The alphameric data AA that are read in and written out immediately before P1 is read in introduce P1. Here,

$$\mathrm{P1(I)}=10^7p_1(-\zeta)\ \text{for}\ -\zeta=0.1*(\mathrm{I}-1),\ \ \mathrm{I}=1,2,\cdots,76 \tag{5.10}$$

$$\mathrm{P1(I)}=10^7p_1\left(\frac{1}{\xi^2}\right)\ \text{for}\ \xi=0.02*(\mathrm{I}-77),\ \mathrm{I}=77,78,\cdots,96 \tag{5.11}$$

where $p_1(-\zeta)$ is the parameter p_1 that appears in eq. (B.5) of [2]. The argument $-\zeta$ is given by eq. (B.6) of [2]. The alphameric data AA that are read in and written out immediately before P1D2 is read in introduce P1D2. Here,

$$\mathrm{P1D2(I)}=\delta_m^2,\ \ \mathrm{I}=1,2,\cdots,96 \tag{5.12}$$

where $\{\delta_m^2,\ \mathrm{I}=1,2,\cdots,76\}$ are the modified second differences in the interpolation formula (5.4) for $10^7p_1(-\zeta)$ and $\{\delta_m^2,\ \mathrm{I}=77,78,\cdots,96\}$ are the modified second differences in the interpolation formula (5.4) for $10^7p_1\left(\frac{1}{\xi^2}\right)$ where ξ is given by (5.8). Precise definitions of the interpolation formulas (5.4) for $10^7p_1(-\zeta)$ and (5.4) for $10^7p_1\left(\frac{1}{\xi^2}\right)$ can be obtained by replacing $10^9z(-\zeta)$ and $10^9\left(z\left(\frac{1}{\xi^2}\right)-\frac{2}{3}\xi^{-3}\right)$ by $10^7p_1(-\zeta)$ and $10^7p_1\left(\frac{1}{\xi^2}\right)$, respectively, in the definitions of the interpolation formulas (5.4) for $10^9z(-\zeta)$ and (5.4) for $10^9\left(z\left(\frac{1}{\xi^2}\right)-\frac{2}{3}\xi^{-3}\right)$. The latter definitions are contained in the five sentences that follow (5.8).

The alphameric data that are read in and written out immediately before P2 is read in introduce P2. Here,

$$\mathrm{P2(I)}=10^5p_2(-\zeta)\ \text{for}\ -\zeta=0.1*(\mathrm{I}-1),\ \mathrm{I}=1,2,\cdots,76 \tag{5.13}$$

where $p_2(-\zeta)$ is the parameter p_2 that appears in eq. (B.5) of [2]. It is assumed that

$$p_2(-\zeta) = 0 \text{ for } -\zeta > 7.5. \tag{5.14}$$

5.1.4 Interpolation Data for Roots of Derivatives of Bessel Functions of Large Order

In this section, interpolation data are obtained for the parameters q_1, q_2, and q_3 that appear in eq. (B.17) of [2]. The parameter z in eq. (B.17) of [2] is the same function of $-\zeta$ as in eq. (B.5) of [2]. Interpolation data for z was obtained in Section 5.1.3.

The alphameric data AA that are read in and written out immediately before Q1 is read in introduce Q1. Here,

$$\mathrm{Q1(I)} = 10^7(-\zeta)q_1(-\zeta) \text{ for } -\zeta = 0.1*(\mathrm{I}-1),\ \mathrm{I} = 1,2,\cdots,11 \tag{5.15}$$

$$\mathrm{Q1(I)} = 10^7 q_1(-\zeta) \text{ for } -\zeta = 0.1*(\mathrm{I}-2),\ \mathrm{I} = 12,13,\cdots,77 \tag{5.16}$$

$$\mathrm{Q1(I)} = 10^7 q_1\left(\frac{1}{\xi^2}\right) \text{ for } \xi = 0.02*(\mathrm{I}-78),\ \mathrm{I} = 78,79,\cdots,97 \tag{5.17}$$

where $q_1(-\zeta)$ is the parameter q_1 that appears in eq. (B.17) of [2]. The argument $-\zeta$ is given by eq. (B.6) of [2]. The alphameric data AA that are read in and written out immediately before Q1D2 is read in introduce Q1D2. Here,

$$\mathrm{Q1D2(I)} = \delta_m^2,\ \mathrm{I} = 1,2,\cdots,97 \tag{5.18}$$

where $\{\delta_m^2, \mathrm{I} = 1,2,\cdots,11\}$ are the modified second differences in the interpolation formula (5.4) for $10^7(-\zeta)q_1(-\zeta)$ of (5.15), $\{\delta_m^2, \mathrm{I} = 12,13,\cdots,77\}$ are the modified second differences in (5.4) for $10^7 q_1(-\zeta)$ of (5.16), and $\{\delta_m^2, \mathrm{I} = 78,79,\cdots,97\}$ are the modified second differences in (5.4) for $10^7 q_1\left(\frac{1}{\xi^2}\right)$ of (5.17). The alphameric data AA that are read in and written out immediately before Q1D4 is read in introduce Q1D4. Here,

$$\mathrm{Q1D4(I)} = \gamma^4,\ \mathrm{I} = 1,2,\cdots,17 \tag{5.19}$$

where $\{\gamma^4, \mathrm{I} = 1,2,\cdots,17\}$ are the modified fourth differences that complement the modified second differences $\{\delta_m^2, \mathrm{I} = 1,2,\cdots,17\}$ in (5.18). The remaining modified second differences $\{\delta_m^2, \mathrm{I} = 18,19,\cdots,97\}$ in (5.18) are not complemented by any modified fourth differences.

The alphameric data AA that are read in and written out immediately before Q2 is read in introduce Q2. Here,

$$Q2(I) = 10^6\,(-\zeta)^3\,q_2(-\zeta) \text{ for } -\zeta = 0.1*(I-1),\ I = 1,2,\cdots,11 \quad (5.20)$$

$$Q2(I) = 10^5 q_2\,(-\zeta) \text{ for } -\zeta = 0.1*(I-2),\ I = 12,13,\cdots,50 \quad (5.21)$$

where $q_2(-\zeta)$ is the parameter q_2 that appears in eq. (B.17) of [2]. It is assumed that

$$q_2(-\zeta) = 0 \text{ for } -\zeta > 4.8. \quad (5.22)$$

The alphameric data that are read in and written out immediately before Q2D2 are read in introduce Q2D2. Here,

$$Q2D2(I) = \delta_m^2,\ I = 1,2,\cdots,30 \quad (5.23)$$

where $\{\delta_m^2, I = 1,2,\cdots,11\}$ are the modified second differences in the interpolation formula (5.4) for $10^6(-\zeta)^3 q_2(-\zeta)$ of (5.20), and $\{\delta_m^2, I = 12,13,\cdots, 30\}$ are the modified second differences in (5.4) for $10^5 q_2(-\zeta)$ of (5.21). There are no modified second differences for $-\zeta > 2.8$ in (5.21).

The alphameric data that are read in and written out immediately before Q3 is read in introduce Q3. Here,

$$Q3(I) = 10^5(-\zeta)^5 q_3(-\zeta) \text{ for } -\zeta = 0.1*(I-1),\ I = 1,2,\cdots,11 \quad (5.24)$$

where $q_3(-\zeta)$ is the parameter q_3 that appears in eq. (B.17) of [2]. It is assumed that

$$q_3(-\zeta) = 0 \text{ for } -\zeta > 1.0. \quad (5.25)$$

5.1.5 Negative Roots of the Airy Function and Its Derivative

The alphameric data AA that are read in and written out immediately before A is read in introduce A. Here,

$$A(S) = a_S \text{ for } S = 1,2,\cdots,50 \quad (5.26)$$

where a_S is the S^{th} negative root of the Airy function Ai (see eq. (B.7) of [2]). The alphameric data AA that are read in and written out immediately before AP is read in introduce AP. Here,

$$AP(S) = a'_S \text{ for } S = 1,2,\cdots,50 \quad (5.27)$$

where a'_S is the S^{th} negative root of Ai′ where Ai is the derivative of the Airy function Ai (see eq. (B.19) of [2]). After AP is written out, PI3, PI4, and TT are defined by

$$\mathrm{PI3} = \frac{3\pi}{8} \tag{5.28}$$

$$\mathrm{PI4} = \frac{\pi}{4} \tag{5.29}$$

$$\mathrm{TT} = \frac{2}{3}. \tag{5.30}$$

The variables PI4 and TT are common variables that will be used in the subroutine BES. The variables PI3 and TT are used in DO loop 25.

DO loop 25 calculates a_s of (B.9) of [2] and a'_s of (B.21) of [2] and stores them in A(s) and AP(s), respectively, for $\{s = 50, 51, \cdots, 200\}$. According to what is stated in the sentence containing eq. (B.9) of [2] and the sentence containing eq. (B.21) of [2], we should have started s at 51 rather than 50. We started s at 50 to obtain calculated values of a_{50} and a'_{50} that we could compare with the previously read in values a_{50} of (5.26) and a'_{50} of (5.27). For this comparison, see the last paragraph of Section 2.2.2. In DO loop 25,

$$\mathrm{AM} = \lambda \tag{5.31}$$

$$\mathrm{UM} = \mu \tag{5.32}$$

$$\mathrm{A(S)} = a_S \tag{5.33}$$

$$\mathrm{AP(S)} = a'_S \tag{5.34}$$

where λ is the right-hand side of eq. (B.10) of [2] when s = S and μ is the right-hand side of eq. (B.22) of [2] when s = S. Furthermore, a_S is the right-hand side of eq. (B.9) of [2] when s = S and a'_S is the right-hand side of eq. (B.21) of [2] when s = S.

5.2 Listing of the Subroutine BESIN

```
      SUBROUTINE BESIN
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /PI/PI
      COMMON /BESIN/X(21,50),XP(21,50),Z(96),ZD2(96),ZD4(96),
     1P1(96).P1D2(96),P2(76),Q1(97),Q1D2(97),Q1D4(17),Q2(50),
```

```
 2Q2D2(30),Q3(11),A(200),AP(200),PI4,TT
  INTEGER S
  REAL*4 AA(40)
  OPEN(UNIT=22,FILE='BESIN.DAT', STATUS='OLD')
  OPEN(UNIT=23,FILE='BESOUT.DAT', STATUS='OLD')
  READ(22,10)(AA(I),I=1,20)
10 FORMAT(20A4)
  WRITE(23,10)(AA(I),I=1,20)
  DO 12 N=1,21
  READ(22,13)(X(N,S),S=1,50)
13 FORMAT(5F13.8)
12 CONTINUE
  DO 11 J=1,4
  J1=(J-1)*5+1
  J2=J1+4
  WRITE(23,22)((X(N,S),N=J1,J2),S=1,50)
22 FORMAT(5F13.8)
11 CONTINUE
  WRITE(23,23)(X(21,S),S=1,50)
23 FORMAT(F13.8)
  READ(22,10)(AA(I),I=1,20)
  WRITE(23,10)(AA(I),I=1,20)
  DO 14 N=1,21
  READ(22,13)(XP(N,S),S=1,50)
14 CONTINUE
  DO 24 J=1,4
  J1=(J-1)*5+1
  J2=J1+4
  WRITE(23,22)((XP(N,S),N=J1,J2),S=1,50)
24 CONTINUE
  WRITE(23,23)(XP(21,S),S=1,50)
  READ(22,10)(AA(I),I=1,40)
  WRITE(23,10)(AA(I),I=1,40)
  READ(22,15)(Z(I),I=1,96)
15 FORMAT(5F13.0)
  WRITE(23,15)(Z(I),I=1,96)
  READ(22,10)(AA(I),I=1,20)
  WRITE(23,10)(AA(I),I=1,20)
  READ(22,16)(ZD2(I),I=1,96)
16 FORMAT(5F9.0)
  WRITE(23,16)(ZD2(I),I=1,96)
  READ(22,10)(AA(I),I=1,20)
  WRITE(23,10)(AA(I),I=1,20)
  READ(22,17)(ZD4(I),I=1,96)
```

```
 17 FORMAT(5F3.0)
    WRITE(23,17)(ZD4(I),I=1,96)
    READ(22,10)(AA(I),I=1,20)
    WRITE(23,10)(AA(I),I=1,20)
    READ(22,18)(P1(I),I=1,96)
 18 FORMAT(5F8.0)
    WRITE(23,18)(P1(I),I=1,96)
    READ(22,10)(AA(I),I=1,20)
    WRITE(23,10)(AA(I),I=1,20)
    READ(22,19)(P1D2(I),I=1,96)
 19 FORMAT(5F5.0)
    WRITE(23,19)(P1D2(I),I=1,96)
    READ(22,10)(AA(I),I=1,20)
    WRITE(23,10)(AA(I),I=1,20)
    READ(22,19)(P2(I),I=1,76)
    WRITE(23,19)(P2(I),I=1,76)
    READ(22,10)(AA(I),I=1,20)
    WRITE(23,10)(AA(I),I=1,20)
    READ(22,16)(Q1(I),I=1,97)
    WRITE(23,16)(Q1(I),I=1,97)
    READ(22,10)(AA(I),I=1,20)
    WRITE(23,10)(AA(I),I=1,20)
    READ(22,20)(Q1D2(I),I=1,97)
 20 FORMAT(5F7.0)
    WRITE(23,20)(Q1D2(I),I=1,97)
    READ(22,10)(AA(I),I=1,20)
    WRITE(23,10)(AA(I),I=1,20)
    READ(22,17)(Q1D4(I),I=1,17)
    WRITE(23,17)(Q1D4(I),I=1,17)
    READ(22,10)(AA(I),I=1,20)
    WRITE(23,10)(AA(I),I=1,20)
    READ(22,20)(Q2(I),I=1,50)
    WRITE(23,20)(Q2(I),I=1,50)
    READ(22,10)(AA(I),I=1,20)
    WRITE(23,10)(AA(I),I=1,20)
    READ(22,19)(Q2D2(I),I=1,30)
    WRITE(23,19)(Q2D2(I),I=1,30)
    READ(22,10)(AA(I),I=1,20)
    WRITE(23,10)(AA(I),I=1,20)
    READ(22,19)(Q3(I),I=1,11)
    WRITE(23,19)(Q3(I),I=1,11)
    READ(22,10)(AA(I),I=1,20)
    WRITE(23,10)(AA(I),I=1,20)
    READ(22,21)(A(S),S=1,50)
```

```
21 FORMAT(5F12.8)
   WRITE(23,21)(A(S),S=1,50)
   READ(22,10)(AA(I),I=1,20)
   WRITE(23,10)(AA(I),I=1,20)
   READ(22,21)(AP(S),S=1,50)
   WRITE(23,21)(AP(S),S=1,50)
   PI3=0.375D+0*PI
   PI4=PI/4.D+0
   TT=2.D+0/3.D+0
   DO 25 S=50,200
   NS=4*S
   AM=PI3*(NS-1)
   UM=PI3*(NS-3)
   AM2=AM*AM
   UM2=UM*UM
   A(S)=-AM**TT*(1.D+0+.104166666667D+0/AM2)
   AP(S)=-UM**TT*(1.D+0-.145833333333D+0/UM2)
25 CONTINUE
   WRITE(23,26) A(50),AP(50)
26 FORMAT('A(50)=',F12.8,', AP(50)=',F12.8)
   RETURN
   END
```

Chapter 6

The Subroutine BES

The subroutine BES(N,XJ,XJP)† sets

$$\text{XJ(S)} = j_{ns} \tag{6.1}$$

$$\text{XJP(S)} = j'_{ns} \tag{6.2}$$

where

$$n = \text{N} - 1 \tag{6.3}$$

$$s = \text{S for S} = 1, 2, \cdots, \text{S}_{\mathbf{max}}. \tag{6.4}$$

Here, j_{ns} is the s^{th} root of the Bessel function J_n and j'_{ns} is the s^{th} root of J'_n (see Appendix B of [2]). In (6.4), $\text{S}_{\mathbf{max}}$ is the smallest integer s such that

$$j'_{ns} > \text{XM} \tag{6.5}$$

where XM is an input variable.

In the common block labeled BES, XM is the input variable mentioned in the previous sentence, and SMAX is an output variable. As calculated in the subroutine BES,

$$\text{SMAX} = s_{\mathbf{max}} \tag{6.6}$$

where $s_{\mathbf{max}}$ is, as defined in Appendix B of [2], the largest integer s such that

$$\left.\begin{array}{ll} j_{0,s} \le \text{XM}, & n = 0 \\ j'_{ns} \le \text{XM}, & n = 1, 2, \cdots \end{array}\right\}. \tag{6.7}$$

†See the listing of the subroutine BES in Section 6.3.

If $n = 0$ and $j_{0,1} > \text{XM}$ or if $n \neq 0$ and $j'_{n1} > \text{XM}$, then the subroutine BES sets SMAX = 0. The common block labeled BESIN contains input data obtained by running the subroutine BESIN. This input data is described in Chapter 5. The subroutine BES calls the subroutine INTERPOL. The subroutines BES and INTERPOL communicate by means of the variables in the common block labeled INTERPOL. The subroutine INTERPOL uses P and I to calculate IP, CP, E2, F2, M4, N4, and AZ (see Chapter 7).

6.1 The Roots j_{ns} and j'_{ns} for $n \leq 19$

If $n \leq 19$ where n is given by (6.3), then the statement

IF(N.GT.20) GO TO 11

at the beginning of the subroutine BES allows execution to proceed to DO loop 12 and, if necessary, to DO loop 14. In these DO loops, XJ(S) and XJP(S) of (6.1) and (6.2) are obtained for $\{S = 1, 2, \cdots, S_{max}\}$ where S_{max} is defined by means of (6.5). Thanks to the branch statement

IF(XJP(S).GT.XM) GO TO 13

before statement 12 and the branch statement

IF(XJP(S).GT.XM) GO TO 13

before statement 14, $S = S_{max}$ immediately before execution of statement 13 provided that

$$j'_{n,200} > \text{XM}. \tag{6.8}$$

If (6.8) is not satisfied, then execution terminates on the statement

STOP 14

after statement 14. Statement 13 and the statement following it use S_{max} to set

$$\text{SMAX} = s_{max} \tag{6.9}$$

where s_{max} is defined by means of (6.7). If $n = 0$ and $j_{ns} \leq \text{XM}$ when $s = S_{max}$, then

$$s_{max} = S_{max}\,. \tag{6.10}$$

Otherwise,

$$s_{max} = S_{max} - 1. \tag{6.11}$$

6.1.1 Tabulated Values of j_{ns} and j'_{ns} for $s = 1, 2, \cdots, 49$

In DO loop 12, X(N, S) and XP(N, S) are, respectively, the values of $j_{N-1,S}$ and $j'_{N-1,S}$ taken from Tables I and II of [6]. The "outer fringe" tabulated values {X(21, S) and XP(21, S), S = 1, 2, ⋯, 50} and {X(N, 50) and XP(N, 50), N = 1, 2, ⋯, 20} were not used in DO loop 12. Values of $\{j_{20,s}$ and $j'_{20,s}$, $s = 1, 2, \cdots, 50\}$ and $\{j_{n,50}$ and $j'_{n,50}$, $n = 0, 1, \cdots, 19\}$ were computed later in the subroutine BES for comparison with the tabulated values {X(21, S) and XP(21, S), S = 1, 2, ⋯, 50} and {X(N, 50) and XP(N, 50), N = 1, 2, ⋯, 20}. To obtain this comparison, we had to increase the value of the input variable XM from 40 (see (2.53)) to 190. We also had to insert statements to write out the computed values of {X(21, S) and XP(21, S), S = 1, 2, ⋯, 50} and {X(N, 50) and XP(N, 50), N = 1, 2, ⋯, 20}. We made these changes and ran the computer program to obtain output data not shown in the present report. In these output data, each of the computed values of X(21, 1), {X(21, S), XP(21, S), S = 2, 3, ⋯, 50} and {X(N, 50), XP(N, 50), N = 1, 2, ⋯, 20} was within 3×10^{-8} of the tabulated value. The difference between the computed and tabulated values of XP(21, 1) was a bit larger. The computed value of XP(21, 1) was 22.2194671 as opposed to the tabulated value of 22.21914648.

6.1.2 Truncated Expansions for j_{ns} and j'_{ns} for $s \geq 50$

If $j'_{n,49} \leq$ XM, then DO loop 12 terminates normally and control passes to the statement that follows statement 12. As a result, DO loop 14 and the 16 statements prior to it are executed. In DO loop 14,

$$\mathrm{XJ(S)} = j_{ns} \text{ for } n = \mathrm{N} - 1 \text{ and } s = \mathrm{S} \tag{6.12}$$

$$\mathrm{XJP(S)} = j'_{ns} \text{ for } n = \mathrm{N} - 1 \text{ and } s = \mathrm{S} \tag{6.13}$$

where j_{ns} and j'_{ns} are, respectively, calculated values of the right-hand sides of eqs. (B.23) and (B.24) of [2].

The 17 statements prior to DO loop 14 set

$$\mathrm{NA} = n \tag{6.14}$$

$$\mathrm{N2} = 2n - 1 \tag{6.15}$$

$$\mathrm{N3} = \begin{cases} 2n-1, & n=0 \\ 2n-3, & n \neq 0 \end{cases} \tag{6.16}$$

$$\mathrm{UM} = \mu \tag{6.17}$$

$$\mathrm{UM1} = \mu - 1 \tag{6.18}$$

$$\mathrm{A1} = \frac{A_1}{8} \tag{6.19}$$

$$\mathrm{A3} = \frac{A_3}{384} \tag{6.20}$$

$$\mathrm{UM2} = \mu^2 \tag{6.21}$$

$$\mathrm{A5} = \frac{A_5}{61440} \tag{6.22}$$

$$\mathrm{UM3} = \mu^3 \tag{6.23}$$

$$\mathrm{A7} = \frac{A_7}{20643840} \tag{6.24}$$

$$\mathrm{AP1} = \frac{A_1'}{8} \tag{6.25}$$

$$\mathrm{AP3} = \frac{A_3'}{384} \tag{6.26}$$

$$\mathrm{AP5} = \frac{a_5'}{61440} \tag{6.27}$$

$$\mathrm{UM4} = \mu^4 \tag{6.28}$$

$$\mathrm{AP7} = \frac{a_7'}{20643840} \tag{6.29}$$

where

$$\mu = 4n^2. \tag{6.30}$$

Moreover, A_1, A_3, A_5, A_7, A_1', A_3', A_5', and A_7' are given, respectively, by eqs. (B.26)–(B.29) and (B.32)–(B.35) of [2]. As given by (6.19), (6.20), (6.22), and (6.24), A1, A3, A5, and A7 are, respectively, the coefficients of the $1/\beta$, $1/\beta^3$, $1/\beta^5$, and $1/\beta^7$ terms inside the summation sign in eq. (B.23) of [2]. As given by (6.25)–(6.27), and (6.29), AP1, AP3, AP5, and AP7 are, respectively, the coefficients of the $1/\beta'$, $1/\beta'^3$, $1/\beta'^5$, and $1/\beta'^7$ terms inside the summation sign in eq. (B.24) of [2].

In DO loop 14,

$$\mathrm{NS} = 4s \tag{6.31}$$

$$\mathrm{B} = \beta \tag{6.32}$$
$$\mathrm{B2} = \beta^2 \tag{6.33}$$
$$\mathrm{B3} = \beta^3 \tag{6.34}$$
$$\mathrm{B5} = \beta^5 \tag{6.35}$$
$$\mathrm{XJ(S)} = j_{ns} \tag{6.36}$$
$$\mathrm{BP} = \beta' \tag{6.37}$$
$$\mathrm{BP2} = \beta'^2 \tag{6.38}$$
$$\mathrm{BP3} = \beta'^3 \tag{6.39}$$
$$\mathrm{BP5} = \beta'^5 \tag{6.40}$$
$$\mathrm{XJP(S)} = j'_{ns} \tag{6.41}$$

where β, j_{ns}, and j'_{ns} are given, respectively, by eqs. (B.25), (B.23), and (B.24) of [2]. Equation (B.31) of [2] is not correct when $n = 0$; the value of β' in (6.37) is given by[†]

$$\beta' = \begin{cases} (2n + 4s + 1)\pi/4, & n = 0 \\ (2n + 4s - 3)\pi/4, & n \neq 0. \end{cases} \tag{6.42}$$

6.2 The Roots j_{ns} and j'_{ns} for $n \geq 20$

If $n \geq 20$ where n is given by (6.3), then the statement

IF(N.GT.20) GO TO 11

at the beginning of the subroutine BES sends execution to statement 11. Control eventually passes to DO loop 15, which sets

$$\mathrm{XJ(S)} = j_{ns} \text{ for } n = \mathrm{N} - 1 \text{ and } s = \mathrm{S} \tag{6.43}$$
$$\mathrm{XJP(S)} = j'_{ns} \text{ for } n = \mathrm{N} - 1 \text{ and } s = \mathrm{S} \tag{6.44}$$

where j_{ns} and j'_{ns} are, respectively, calculated values of the right-hand sides of eqs. (B.5) and (B.17) of [2]. The six statements before DO loop 15 set

$$\mathrm{CN} = n \tag{6.45}$$
$$\mathrm{CN1} = 10^{-9} n \tag{6.46}$$

[†]Since our $j'_{o,s}$ is that of [6] with s replaced by $s + 1$, our β' is that of [6] with s replaced by $s + 1$ when $n = 0$.

$$\mathrm{CN2} = \frac{10^{-7}}{n} \tag{6.47}$$

$$\mathrm{CN3} = \frac{10^{-5}}{n^3} \tag{6.48}$$

$$\mathrm{CN4} = \frac{10^{-5}}{n^5} \tag{6.49}$$

$$\mathrm{CNZ} = -n^{-2/3}. \tag{6.50}$$

6.2.1 Calculation of j_{ns} for $n \geq 20$

The group of statements before statement 17 in DO loop 15 calculates j_{ns} of (6.43). The first of these statements sets

$$\mathrm{ZETA} = -\zeta \tag{6.51}$$

where $-\zeta$ is given by eq. (B.6) of [2].

The group of eight statements after the branch statement

IF(ZETA.GT.7.5D+0) GO TO 16

implements (6.43) when $-\zeta < 7.5$. The third of these statements sets

$$\mathrm{P} = p \tag{6.52}$$

where p appears in (5.4). The fourth of these statements sets I such that Z(I) is the value of $10^9 z$ at the nearest smaller tabulated value of $-\zeta$. The fifth of these statements calculates the computer program variables IP, CP, E2, F2, M4, N4, and AZ. These variables are defined by (7.8)-(7.14), respectively. The sixth and seventh of these statements set

$$\mathrm{AP1} = 10^7 p_1 \tag{6.53}$$

$$\mathrm{AP2} = 10^5 p_2 \tag{6.54}$$

where $10^7 p_1$ is the right-hand side of (5.4) for $10^7 p_1(-\zeta)$ and $10^5 p_2$ is the right-hand side of (5.4) for $10^5 p_2(-\zeta)$. Here, $p_1(-\zeta)$ and $p_2(-\zeta)$ are the interpolated values of p_1 and p_2 in (B.5) of [2]. The eighth of these statements sets XJ(S) equal to j_{ns} of eq. (B.5) of [2] when $-\zeta < 7.5$.

Statement 16 and the seven statements following it implement (6.43) when $-\zeta \geq 7.5$. The first of these statements sets

$$\mathrm{XI} = \xi \tag{6.55}$$

where ξ is given by (5.8). The fourth of these statements sets

$$P = p \tag{6.56}$$

where p appears in (5.4). The fifth of these statements sets I such that Z(I) is the value of $10^9\left(z\left(\frac{1}{\xi^2}\right) - \frac{2}{3}\xi^{-3}\right)$ at the nearest smaller tabulated value of ξ. The sixth of these statements calculates the computer program variables IP, CP, E2, F2, M4, N4, and AZ. These variables are defined by (7.8)-(7.13) and (7.15), respectively. The seventh of these statements sets

$$\mathrm{AP1} = 10^7 p_1 \tag{6.57}$$

where $10^7 p_1$ is the right-hand side of (5.4) for $10^7 p_1\left(\frac{1}{\xi^2}\right)$. It is assumed that $p_2\left(\frac{1}{\xi^2}\right) = 0$. The eighth of these statements sets XJ(S) equal to j_{ns} of eq. (B.5) of [2] when $-\zeta \geq 7.5$.

6.2.2 Calculation of j'_{ns} for $n \geq 20$

Statement 17 and all statements between statements 17 and 20 in DO loop 15 calculate j'_{ns} of (6.44). Statement 17 sets

$$\mathrm{ZETA} = -\zeta \tag{6.58}$$

where $-\zeta$ is given by eq. (B.18) of [2].

The group of 18 statements after the branch statement

IF(ZETA.GE.7.5D+0) GO TO 18

implements (6.44) when $-\zeta < 7.5$. The third of these statements sets

$$P = p \tag{6.59}$$

where p appears in (5.4). The fourth of these statements sets I such that Z(I) is $10^9 z(-\zeta)$ at the nearest smaller tabulated value of $-\zeta$. The fifth of these statements calculates the computer program variables IP, CP, E2, F2, M4, N4, and AZ. These variables are defined by (7.8)-(7.14), respectively. The sixth of these statements adds 1 to I if I $\geq$ 11. This is necessary because I = 11 in (5.15) and I = 12 in (5.16) both indicate the same argument $-\zeta = 1.0$. The eighth through seventeenth of these statements set

$$\mathrm{AQ1} = 10^7 q_1(-\zeta) \tag{6.60}$$

$$\mathrm{AQ2} = 10^5 q_2(-\zeta) \tag{6.61}$$

$$\mathrm{AQ3} = 10^5 q_3(-\zeta) \tag{6.62}$$

where $q_1(-\zeta)$, $q_2(-\zeta)$, and $q_3(-\zeta)$ are the interpolated values of the quantities q_1, q_2, and q_3 in eq. (B.17) of [2].

The formulas for AQ1, AQ2, and AQ3 that were programmed in the subroutine BES can be obtained by first substituting P, CP, E2, F2, M4, and N4† for p, $1-p$, E_2, F_2, M_4, and N_4 in the interpolation formula (5.4) and then applying the resulting interpolation formula to the data in Section 5.1.4. Formula (5.4) for $10^7(-\zeta)q_1(-\zeta)$ of (5.15) gives

$$\begin{aligned} \mathrm{AQ1} = & \frac{\mathrm{CP} * \mathrm{Q1(I)} + \mathrm{P} * \mathrm{Q1(IP)} + \mathrm{E2} * \mathrm{Q1D2(I)} + \mathrm{F2} * \mathrm{Q1D2(IP)}}{-\zeta} \\ & + \frac{\mathrm{M4} * \mathrm{Q1D4(I)} + \mathrm{N4} * \mathrm{Q1D4(IP)}}{-\zeta} \quad \text{for } 0 \leq -\zeta < 1.0 \end{aligned} \tag{6.63}$$

where, as in the program,

$$\mathrm{IP} = \mathrm{I} + 1. \tag{6.64}$$

Formula (5.4) for $10^7 q_1(-\zeta)$ of (5.16) gives

$$\begin{aligned} \mathrm{AQ1} = & \ \mathrm{CP} * \mathrm{Q1(I)} + \mathrm{P} * \mathrm{Q1(IP)} + \mathrm{E2} * \mathrm{Q1D2(I)} + \mathrm{F2} * \mathrm{Q1D2(IP)} \\ & + \mathrm{M4} * \mathrm{Q1D4(I)} + \mathrm{N4} * \mathrm{Q1D4(IP)} \quad \text{for } 1.0 \leq -\zeta < 1.5 \end{aligned} \tag{6.65}$$

and

$$\begin{aligned} \mathrm{AQ1} = & \ \mathrm{CP} * \mathrm{Q1(I)} + \mathrm{P} * \mathrm{Q1(IP)} + \mathrm{E2} * \mathrm{Q1D2(I)} + \mathrm{F2} * \mathrm{Q1D2(IP)} \\ & \text{for } 1.5 \leq -\zeta < 7.5. \end{aligned} \tag{6.66}$$

Formula (5.4) for $10^6(-\zeta)^3 q_2(-\zeta)$ of (5.20) gives

$$\begin{aligned} \mathrm{AQ2} = & \ (\mathrm{CP} * \mathrm{Q2(I)} + \mathrm{P} * \mathrm{Q2(IP)} + \mathrm{E2} * \mathrm{Q2D2(I)} + \mathrm{F2} * \mathrm{Q2D2(IP)}) \\ & * \left(\frac{0.1}{\zeta^3}\right) \quad \text{for } 0 \leq -\zeta < 1.0. \end{aligned} \tag{6.67}$$

Formula (5.4) for $10^5 q_2(-\zeta)$ of (5.21) gives

$$\begin{aligned} \mathrm{AQ2} = & \ \mathrm{CP} * \mathrm{Q2(I)} + \mathrm{P} * \mathrm{Q2(IP)} + \mathrm{E2} * \mathrm{Q2D2(I)} + \mathrm{F2} * \mathrm{Q2D2(IP)} \\ & \text{for } 1.0 \leq -\zeta < 2.8 \end{aligned} \tag{6.68}$$

†See (7.1) and (7.9)–(7.13).

and

$$\text{AQ2} = \text{CP} * \text{Q2(I)} + \text{P} * \text{Q2(IP)} \quad \text{for} \quad 2.8 \leq -\zeta < 4.8. \tag{6.69}$$

It is assumed that

$$\text{AQ2} = 0 \quad \text{for} \quad 4.8 \leq -\zeta < 7.5. \tag{6.70}$$

Note that AQ2 suddenly jumps from a nonzero value calculated from (6.69) to the zero value of (6.70) as $-\zeta$ passes through 4.8. The resulting discontinuity of the interpolated value of $q_2(-\zeta)$ at $-\zeta = 4.8$ does not cause serious error because the actual value of $q_2(4.8)$ is small. Formula (5.4) for $10^5(-\zeta)^5 q_3(-\zeta)$ of (5.24) gives

$$\text{AQ3} = \frac{\text{CP} * \text{Q3(I)} + \text{P} * \text{Q3(IP)}}{-\zeta^5} \quad \text{for} \quad 0 \leq -\zeta < 1.0. \tag{6.71}$$

It is assumed that

$$\text{AQ3} = 0 \quad \text{for} \quad 1.0 \leq -\zeta < 7.5. \tag{6.72}$$

Statement 19 uses CN1, CN2, CN3, CN4, AZ, AQ1, AQ2, and AQ3 of (6.46)–(6.49), (7.14), and (6.60)–(6.62), respectively, to set XJP(S) equal to j'_{ns} of (B.17) of [2]† when $-\zeta < 7.5$.

The group of statements consisting of statement 18 and all statements between statements 18 and 20 in DO loop 15 implements (6.44) when $-\zeta \geq 7.5$. The first of these statements sets

$$\text{XI} = \xi \tag{6.73}$$

where ξ is given by (5.8). The fourth of these statements sets

$$\text{P} = p \tag{6.74}$$

where p appears in (5.4). The fifth of these statements sets I such that Z(I) is $10^9 \left(\text{Z}\left(\frac{1}{\xi^2}\right) - \frac{2}{3}\xi^{-3}\right)$ at the nearest smaller tabulated value of ξ. The sixth of these statements calculates the computer program variables IP, CP, E2, F2, M4, N4, and AZ. These variables are defined by (7.10)–(7.13) and (7.15),

†Here, eq. (B.17) of [2] with $q_2 = q_3 = 0$ is meant rather than (B.17) of [2] as it stands.

respectively. The seventh of these statements adds 1 to I because the value of I in Q1(I) and Q1D2(I)[†] to be used in the ninth of these statements is 1 more than the value of I in Z(I) of (5.6). The ninth of these statements sets

$$AQ1 = 10^7 q_1 \left(\frac{1}{\xi^2}\right) \tag{6.75}$$

where $q_1\left(\frac{1}{\xi^2}\right)$ is the interpolated value of the quantity q_1 in eq. (B.17) of [2]. The formula for AQ1 that was programmed in the subroutine BES can be obtained by substituting into the interpolation formula (5.4) the computer program variables P, CP, E2, F2, Q1(I), and Q1D2(I) of (7.1), (7.9)–(7.11), (5.17) and (5.18), respectively. The tenth of these statements uses CN, AZ, TT, XI, CN2, and AQ1 of (5.30), (6.45), (6.47), (6.73), (6.75), and (7.15), respectively, to set XJP(S) equal to j'_{ns} of eq. (B.17) of [2] when $-\zeta \geq 7.5$.

6.3 Listing of the Subroutine BES

```
      SUBROUTINE BES(N,XJ,XJP)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /BES/XM,SMAX
      COMMON /BESIN/X(21,50),XP(21,50),Z(96),ZD2(96),ZD4(96),
     1P1(96),P1D2(96),P2(76),Q1(97),Q1D2(97),Q1D4(17),Q2(50),
     2Q2D2(30),Q3(11),A(200),AP(200),PI4,TT
      COMMON /INTERPOL/P,I,IP,CP,E2,F2,M4,N4,AZ
      INTEGER S,SMAX
      REAL*8 XJ(200),XJP(200),M4,N4
      IF(N.GT.20) GO TO 11
      DO 12 S=1,49
      XJ(S)=X(N,S)
      XJP(S)=XP(N,S)
      IF(XJP(S).GT.XM) GO TO 13
   12 CONTINUE
      NA=N-1
      N2=2*NA-1
      N3=2*NA-3
      IF(N.EQ.1) N3=N3+4
      UM=4*NA*NA
      UM1=UM-1.D+0
      A1=UM1/8.D+0
```

[†]See (5.17) and (5.18).

```
      A3=UM1*(7.D+0*UM-31.D+0)/384.D+0
      UM2=UM*UM
      A5=UM1*(83.D+0*UM2-982.D+0*UM+3779.D+0)/15360.D+0
      UM3=UM2*UM
      A7=UM1*(6949.D+0*UM3-153855.D+0*UM2+1585743.D+0*UM-6277237.D+0)
     1/3440640.D+0
      AP1=(UM+3.D+0)/8.D+0
      AP3=(7.D+0*UM2+82.D+0*UM-9.D+0)/384.D+0
      AP5=(83.D+0*UM3+2075.D+0*UM2-3039.D+0*UM+3537.D+0)/15360.D+0
      UM4=UM3*UM
      AP7=(6949.D+0*UM4+296492.D+0*UM3-1248002.D+0*UM2+7414380.D+0*UM
     1-5853627.D+0)/3440640.D+0
      DO 14 S=50,200
      NS=4*S
      B=PI4*(N2+NS)
      B2=B*B
      B3=B2*B
      B5=B3*B2
      XJ(S)=B-A1/B-A3/B3-A5/B5-A7/(B5*B2)
      BP=PI4*(N3+NS)
      BP2=BP*BP
      BP3=BP2*BP
      BP5=BP3*BP2
      XJP(S)=BP-AP1/BP-AP3/BP3-AP5/BP5-AP7/(BP5*BP2)
      IF(XJP(S).GT.XM) GO TO 13
   14 CONTINUE
      STOP 14
   11 CN=N-1
      CN1=1.D-9*CN
      CN2=1.D-7/CN
      CN3=1.D-5/CN**3
      CN4=1.D-5/CN**5
      CNZ=-CN**(-TT)
      DO 15 S=1,200
      ZETA=CNZ*A(S)
      IF(ZETA.GE.7.5D+0) GO TO 16
      Z10=10.D+0*ZETA
      I=Z10
      P=Z10-I
      I=I+1
      CALL INTERPOL
      AP1=CP*P1(I)+P*P1(IP)+E2*P1D2(I)+F2*P1D2(IP)
      AP2=CP*P2(I)+P*P2(IP)
      XJ(S)=CN1*AZ+CN2*AP1+CN3*AP2
```

```
      GO TO 17
   16 XI=1.D+0/DSQRT(ZETA)
      XI50=50.D+0*XI
      I=XI50
      P=XI50-I
      I=I+77
      CALL INTERPOL
      AP1=CP*P1(I)+P*P1(IP)+E2*P1D2(I)+F2*P1D2(IP)
      XJ(S)=CN*(1.D-9*AZ+TT/XI**3)+CN2*AP1
   17 ZETA=CNZ*AP(S)
      IF(ZETA.GE.7.5D+0) GO TO 18
      Z10=10.D+0*ZETA
      I=Z10
      P=Z10-I
      I=I+1
      CALL INTERPOL
      IF(I.GE.11) I=I+1
      IP=I+1
      AQ1=CP*Q1(I)+P*Q1(IP)+E2*Q1D2(I)+F2*Q1D2(IP)
      IF(I.LE.16) AQ1=AQ1+M4*Q1D4(I)+N4*Q1D4(IP)
      AQ2=0.D+0
      IF(I.LE.49) AQ2=CP*Q2(I)+P*Q2(IP)
      IF(I.LE.29) AQ2=AQ2+E2*Q2D2(I)+F2*Q2D2(IP)
      AQ3=0.D+0
      IF(I.GE.12) GO TO 19
      AQ1=AQ1/ZETA
      AQ2=.1D+0*AQ2/ZETA**3
      AQ3=(CP*Q3(I)+P*Q3(IP))/ZETA**5
   19 XJP(S)=CN1*AZ+CN2*AQ1+CN3*AQ2+CN4*AQ3
      GO TO 20
   18 XI=1.D+0/DSQRT(ZETA)
      XI50=50.D+0*XI
      I=XI50
      P=XI50-I
      I=I+77
      CALL INTERPOL
      I=I+1
      IP=I+1
      AQ1=CP*Q1(I)+P*Q1(IP)+E2*Q1D2(I)+F2*Q1D2(IP)
      XJP(S)=CN*(1.D-9*AZ+TT/XI**3)+CN2*AQ1
   20 IF(XJP(S).GT.XM) GO TO 13
   15 CONTINUE
      STOP 15
   13 SMAX=S-1
      IF(N.EQ.1.AND.XJ(S).LE.XM) SMAX=S
      RETURN
      END
```

Chapter 7

The Subroutine INTERPOL

In the subroutine INTERPOL, the interpolation formula (5.4) is applied to the data in the array Z.

7.1 Description of the Subroutine INTERPOL

In the common block labeled INTERPOL in the subroutine INTERPOL,† P and I are input variables and IP, CP, E2, F2, M4, N4, and AZ are output variables. In the common block labeled BESIN, Z, ZD2, and ZD4 are input variables. The remaining variables in the common block labeled BESIN are not used in the subroutine INTERPOL; these variables connot be removed because they are used in the subroutines BESIN and BES.

The input variables P, Z, ZD2, and ZD4 are defined in terms of variables on the right-hand side of (5.4) as

$$\mathrm{P} = p \tag{7.1}$$
$$\mathrm{Z(I)} = 10^9 f_0 \tag{7.2}$$
$$\mathrm{Z(I+1)} = 10^9 f_1 \tag{7.3}$$
$$\mathrm{ZD2(I)} = \delta^2_{m0} \tag{7.4}$$
$$\mathrm{ZD2(I+1)} = \delta^2_{m1} \tag{7.5}$$
$$\mathrm{ZD4(I)} = \gamma^4_0 \tag{7.6}$$

†The subroutine INTERPOL is listed in Section 7.2.

$$\text{ZD4(I+1)} = \gamma_1^4. \tag{7.7}$$

Because the subscripts 0 or 1 on the right-hand sides of (7.2)–(7.7) denote evaluation at the nearest smaller or nearest larger value of the argument of the function subject to interpolation, the meaning of the input variable I is apparent. The values f_0 and f_1 of the function subject to interpolation are those of $z(-\zeta)$ in (5.5) or $z\left(\frac{1}{\xi^2}\right) - \frac{2}{3}\xi^{-3}$ in (5.6).

The subroutine INTERPOL sets

$$
\begin{aligned}
\text{IP} &= \text{I}+1 && (7.8)\\
\text{CP} &= 1-p && (7.9)\\
\text{E2} &= E_2 && (7.10)\\
\text{F2} &= F_2 && (7.11)\\
\text{M4} &= M_4 && (7.12)\\
\text{N4} &= N_4 && (7.13)\\
\text{AZ} &= 10^9 z(-\zeta), \qquad 0 \le -\zeta < 7.5 && (7.14)\\
\text{AZ} &= 10^9\left(z\left(\frac{1}{\xi^2}\right) - \frac{2}{3}\xi^{-3}\right), \quad 0 \le \xi \le \frac{1}{\sqrt{7.5}} && (7.15)
\end{aligned}
$$

where p appears on the right-hand side of (B.12) of [2]. Moreover, E_2, F_2, M_4, and N_4 are given, respectively, by eqs. (B.13)–(B.16) of [2]. The right-hand side of (7.14) is calculated from the interpolation formula (5.4) for $10^9 z(\zeta)$ when $0 \le -\zeta < 7.5$ and from (5.4) for $10^{-9}\left(\left(\frac{1}{\xi^2}\right) - \frac{2}{3}\xi^{-3}\right)$ when $0 \le \xi \le \frac{1}{\sqrt{7.5}}$. In the subroutine INTERPOL, AZ is the calculated value of the right-hand side of (5.4) with the variables p, $10^l f_0$, $10^l f_1$, δ_{m0}^2, δ_{m1}^2, γ_0^4, γ_1^4, E_2, F_2, M_4, and N_4 replaced by the corresponding variables in the subroutine INTERPOL. These corresponding variables are P, Z(I), Z(I+1), ZD2(I), ZD2(I+1), ZD4(I), ZD4(I+1), E2, F2, M4, and N4, respectively (see (7.1)–(7.7) and (7.10)–(7.13)). In (7.14) and (7.15), z is the interpolated value of z in eq. (B.5) of [2].

7.2 Listing of the Subroutine INTERPOL

```
SUBROUTINE INTERPOL
IMPLICIT REAL*8 (A-H,O-Z)
```

```
      COMMON /INTERPOL/P,I,IP,CP,E2,F2,M4,N4,AZ
      COMMON /BESIN/X(21,50),XP(21,50),Z(96),ZD2(96),ZD4(96),
     1P1(96),P1D2(96),P2(76),Q1(97),Q1D2(97),Q1D4(17),Q2(50),
     2Q2D2(30),Q3(11),A(200),AP(200),PI4,TT
      REAL*8 M4,N4
      IP=I+1
      CP=1.D+0-P
      PP1=P+1.D+0
      PP=P*(P-1.D+0)/6.D+0
      PM2=P-2.D+0
      E2=-PP*PM2
      F2=PP*PP1
      M4=1.D+3*E2*(PP1*(P-3.D+0)/20.D+0+0.184D+0)
      N4=1.D+3*F2*((P+2.D+0)*PM2/20.D+0+0.184D+0)
      AZ=CP*Z(I)+P*Z(IP)+E2*ZD2(I)+F2*ZD2(IP)+M4*ZD4(I)+N4*ZD4(IP)
      RETURN
      END
```

Chapter 8

The Subroutine PHI

The subroutine PHI puts $\phi_p^{(1)}$, $\phi_p^{(2)}$, $\phi_p^{(3)}$, and $\phi_p^{(4)}$ of eqs. (3.40)–(3.43) of [2] in PH1(P), PH2(P), PH3(P), and PH4(P), respectively, for $\{P = 1, 2, \cdots, PMAX\}$ where

$$P = p + 1. \tag{8.1}$$

This is equivalent to putting $\phi^{\alpha 1 \gamma 1}$, $\phi^{\alpha 2 \gamma 1}$, $\phi^{\alpha 1 \gamma 2}$, and $\phi^{\alpha 2 \gamma 2}$ of eqs. (3.32)–(3.35) of [2] in PH1(P), PH3(P), PH2(P), and PH4(P), respectively. The subroutine PHI also puts $\phi^{\alpha 1 \gamma 1}$, $\phi^{\alpha 2 \gamma 1}$, $\phi^{\alpha 1 \gamma 2}$, and $\phi^{\alpha 2 \gamma 2}$ of eqs. (3.36)–(3.39) of [2] in PH1(P+PMAX), PH3(P+PMAX), PH2(P+PMAX), and PH4(P+PMAX), respectively, for $\{P = 1, 2, \cdots, PMAX\}$ where

$$P = m + 1. \tag{8.2}$$

8.1 Description of the Subroutine PHI

In the common block labeled PHI,[†] BX, BX5, PMAX, R, and SGR are input variables, and PH1, PH2, PH3, and PH4 are the output variables appearing in the previous two sentences. The input variable PMAX also appears in these two sentences. The input variables BX, BX5, R, and SGR are defined by (3.24), (3.23), (3.73), and (3.88), respectively. In (3.24), ϕ_o is given by eq. (2.9) of [2] and related to x_o by eq. (2.8) of [2] so that (3.24) can be recast as

$$\text{BX} = \frac{b}{x_o}. \tag{8.3}$$

[†]See the listing of the subroutine PHI in Section 8.2.

Similarly, (3.23) can be recast as

$$\mathrm{BX5} = \frac{b}{2x_o}. \tag{8.4}$$

In the common block labeled PI, PI is given by (3.1).

The index P of DO loop 11 is the integer P that appears in the first two sentences of Chapter 8. The four statements before DO loop 11 set

$$\mathrm{R1} = r \tag{8.5}$$

$$\mathrm{RB} = \frac{rb}{x_o} \tag{8.6}$$

$$\mathrm{SN} = \sin\left(\frac{rb}{x_o}\right) \tag{8.7}$$

$$\mathrm{CS} = \cos\left(\frac{rb}{x_o}\right). \tag{8.8}$$

The first statement in DO loop 11 sets

$$\mathrm{PP} = p\pi, \tag{8.9}$$

The second and third statements in DO loop 11 set

$$\mathrm{AP} = A^+ \tag{8.10}$$

$$\mathrm{AP5} = A^+/2 \tag{8.11}$$

where A^+ is given by eq. (3.44) of [2]. The seven statements immediately before statement 13 set

$$\mathrm{SP} = \begin{cases} 1, & A^+ = 0 \\ \dfrac{\sin A^+}{A^+}, & A^+ \neq 0 \end{cases} \tag{8.12}$$

$$\mathrm{SP5} = \begin{cases} 0, & A^+ = 0 \\ \dfrac{\sin^2(A^+/2)}{(A^+/2)}, & A^+ \neq 0. \end{cases} \tag{8.13}$$

Statement 13 and the statement following it set

$$\mathrm{AM} = A^- \tag{8.14}$$

$$\mathrm{AM5} = A^-/2 \tag{8.15}$$

where A^- is given by eq. (3.45) of [2]. The seven statements immediately before statement 15 set

$$\mathrm{SM} = \begin{cases} 1, & A^- = 0 \\ \dfrac{\sin A^-}{A^-}, & A^- \neq 0 \end{cases} \tag{8.16}$$

$$\mathrm{SM5} = \begin{cases} 0, & A^- = 0 \\ \dfrac{\sin^2(A^-/2)}{(A^+/2)}, & A^- \neq 0. \end{cases} \tag{8.17}$$

Statement 15 and the three statements following it set

$$\mathrm{PH1(P)} = \phi_p^{(1)} \tag{8.18}$$

$$\mathrm{PH2(P)} = \phi_p^{(2)} \tag{8.19}$$

$$\mathrm{PH3(P)} = \phi_p^{(3)} \tag{8.20}$$

$$\mathrm{PH4(P)} = \phi_p^{(4)} \tag{8.21}$$

where $\phi_p^{(1)}$, $\phi_p^{(2)}$, $\phi_p^{(3)}$, and $\phi_p^{(4)}$, are given, respectively, by eqs. (3.40)–(3.43) of [2]. In these equations, it is understood that

$$\frac{\sin A^-}{A^-} \quad \text{be replaced by 1 when} \quad A^- = 0 \tag{8.22}$$

$$\frac{\sin A^+}{A^+} \quad \text{be replaced by 1 when} \quad A^+ = 0 \tag{8.23}$$

$$\frac{\sin^2(A^-/2)}{(A^-/2)} \quad \text{be replaced by 0 when} \quad A^- = 0 \tag{8.24}$$

$$\frac{\sin^2(A^+/2)}{(A^+/2)} \quad \text{be replaced by 0 when} \quad A^+ = 0. \tag{8.25}$$

The four statements prior to statement 11 set

$$\mathrm{PH1(J)} = \phi^{\alpha 1 \gamma 1} \tag{8.26}$$

$$\mathrm{PH3(J)} = \phi^{\alpha 2 \gamma 1} \tag{8.27}$$

$$\mathrm{PH2(J)} = \phi^{\alpha 1 \gamma 2} \tag{8.28}$$

$$\mathrm{PH4(J)} = \phi^{\alpha 2 \gamma 2} \tag{8.29}$$

where

$$J = P + PMAX \tag{8.30}$$

and where $\phi^{\alpha 1\gamma 1}$, $\phi^{\alpha 2\gamma 1}$, $\phi^{\alpha 1\gamma 2}$, and $\phi^{\alpha 2\gamma 2}$ are given, respectively, by eqs. (3.36)–(3.39) of [2].

8.2 Listing of the Subroutine PHI

```
      SUBROUTINE PHI
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /PHI/BX,BX5,PMAX,R,SGR,PH1(100),PH2(100),PH3(100),
     1PH4(100)
      COMMON /PI/PI
      INTEGER R,P,PMAX
      R1=R-1
      RB=R1*BX
      SN=DSIN(RB)
      CS=DCOS(RB)
      DO 11 P=1,PMAX
      PP=(P-1)*PI
      AP=PP+RB
      AP5=.5D+0*AP
      IF(AP.NE.0.D+0) GO TO 12
      SP=1.D+0
      SP5=0.D+0
      GO TO 13
   12 SP=DSIN(AP)/AP
      SP5=DSIN(AP5)
      SP5=SP5*SP5/AP5
   13 AM=PP-RB
      AM5=.5D+0*AM
      IF(AM.NE.0.) GO TO 14
      SM=1.D+0
      SM5=0.D+0
      GO TO 15
   14 SM=DSIN(AM)/AM
      SM5=DSIN(AM5)
      SM5=SM5*SM5/AM5
   15 PH1(P)=BX5*(SM-SP)
      PH2(P)=BX5*(SM5+SP5)
      PH3(P)=BX5*(SP5-SM5)
      PH4(P)=BX5*(SM+SP)
      J=P+PMAX
```

```
      PH1(J)=SGR*(PH2(P)*SN-PH1(P)*CS)
      PH3(J)=SGR*(PH4(P)*SN-PH3(P)*CS)
      PH2(J)=SGR*(PH2(P)*CS+PH1(P)*SN)
      PH4(J)=SGR*(PH4(P)*CS+PH3(P)*SN)
   11 CONTINUE
      RETURN
      END
```

Chapter 9

The Subroutine DGN

The subroutine DGN† calculates $\hat{D}_n^\delta$, $\hat{G}_q^\delta$, $\hat{D}_n^{(3)}$, $\hat{G}_q^{(4)}$, z_{ee}, z_o, z_{oe}, z_{oo}, and $1/\left((n\pi)^2 + \left(\gamma_{rs}^\delta c\right)^2\right)$, respectively, of eqs. (3.82), (3.80), (3.111), (3.109), (3.99)-(3.102) and (3.90) of [2]. Here, as in [2], δ is either TM or TE.

9.1 The Input Variables

The input variables are ITMTE and X in the argument list, S, BKA2, L3, C, C5, and PI5 in the common block labeled DGN, PI in the common block labeled PI, and NMAX in the common block labeled NMAX.

The input variable ITMTE must be either 1 or 2. When ITMTE = 1, the output of the subroutine DGN differs from that when ITMTE = 2 only in that the output variables $\hat{D}_n^{(3)}$ and $\hat{G}_q^{(4)}$ are not calculated when ITMTE = 1. The input variables X and S are such that

$$\mathrm{X(S)} = \begin{cases} x_{rs} & \text{for the calculation of TM quantities} \\ x'_{rs} & \text{for the calculation of TE quantities} \end{cases} \tag{9.1}$$

where x_{rs} and x'_{rs} are defined by (2.8) and (2.11), respectively. Given that $\hat{D}_n^{(3)}$ and $\hat{G}_q^{(4)}$ are strictly TE quantities, we plan to set

$$\mathrm{ITMTE} = \begin{cases} 1 & \text{when } \mathrm{X(S)} = x_{rs} \\ 2 & \text{when } \mathrm{X(S)} = x'_{rs} \end{cases} \tag{9.2}$$

†See the listing of the subroutine DGN in Section 9.4.

when we call the subroutine DGN.

The input variables BKA2, L3, C, C5, PI5, and PI are, as defined by (3.16), (2.5), (2.2), (3.53), (3.14), and (3.1), respectively,

$$\mathrm{BKA2} = (ka)^2 \tag{9.3}$$

$$\mathrm{L3} = \frac{L_3}{a} \tag{9.4}$$

$$\mathrm{C} = \frac{c}{a} \tag{9.5}$$

$$\mathrm{C5} = \frac{c}{2a} \tag{9.6}$$

$$\mathrm{PI5} = \frac{\pi}{2} \tag{9.7}$$

$$\mathrm{PI} = \pi\,. \tag{9.8}$$

The input variable NMAX is such that the output quantities $\hat{D}_n^\delta$, $\hat{D}_n^{(3)}$, and $1/\left((n\pi)^2 + \left(\gamma_{rs}^\delta c\right)^2\right)$ are calculated for $\{n = 0, 1, 2, \cdots, \mathrm{NMAX}-1\}$ and that the output quantities $\hat{G}_q^\delta$ and $\hat{G}_q^{(4)}$ are calculated for $\{q = 0, 1, 2, \cdots, \mathrm{NMAX}-1\}$. Here,

$$\delta = \begin{cases} \mathrm{TM}, & \mathrm{X(S)} = x_{rs} \\ \mathrm{TE}, & \mathrm{X(S)} = x'_{rs}\,. \end{cases} \tag{9.9}$$

9.2 The Output Variables

The output variables are XX, ICUT, GAM, CP, CM, D, G, DQ, GCS, GC2, ZEE, ZZ, ZOE, and ZOO in the argument list, and D3, G4, and PGC in the common block labeled DGN. These variables are defined in terms of variables in [2] by

$$\mathrm{XX} = \begin{cases} x_{rs}^2, & \mathrm{X(S)} = x_{rs} \\ x_{rs}'^2, & \mathrm{X(S)} = x'_{rs} \end{cases} \tag{9.10}$$

$$\mathrm{ICUT} = \begin{cases} 1, & \mathrm{XX} < (ka)^2 \\ 2, & \mathrm{XX} \ge (ka)^2 \end{cases} \tag{9.11}$$

$$\mathrm{GAM} = \begin{cases} \beta_{rs}^\delta a, & \mathrm{XX} < (ka)^2 \\ \gamma_{rs}^\delta a, & \mathrm{XX} \ge (ka)^2 \end{cases} \tag{9.12}$$

$$\mathrm{CP(N)} = n^{\delta+} c \tag{9.13}$$

$$
\begin{aligned}
\mathrm{CM(N)} &= n^{\delta -}c && (9.14)\\
\mathrm{D(N)} &= \hat{D}_n^{\delta} && (9.15)\\
\mathrm{G(N)} &= \hat{G}_n^{\delta} && (9.16)\\
\mathrm{DQ(N)} &= \frac{1}{(n\pi)^2 + (\gamma_{rs}^{\delta}c)^2} && (9.17)\\
\mathrm{GCS} &= \left(\gamma_{rs}^{\delta}c\right)^2 && (9.18)\\
\mathrm{GC2} &= 2\gamma_{rs}^{\delta}c && (9.19)\\
\mathrm{ZEE} &= -4z_{ee} && (9.20)\\
\mathrm{ZZ} &= -4z_{o} && (9.21)\\
\mathrm{ZOE} &= -4z_{oe} && (9.22)\\
\mathrm{ZOO} &= -4z_{oo} && (9.23)\\
\mathrm{D3(N)} &= \hat{D}_n^{(3)} && (9.24)\\
\mathrm{G4(N)} &= \hat{G}_n^{(4)} && (9.25)\\
\mathrm{PGC} &= \frac{\pi}{\gamma_{rs}^{\delta}c}. && (9.26)
\end{aligned}
$$

In (9.11) and (9.12), XX is the computer program variable given by (9.10). In (9.12), δ is given by (9.9), $\beta_{rs}^{\delta}a$ is given by eqs. (3.59) and (3.60) of [2], and $\gamma_{rs}^{\delta}a$ is given by eqs. (3.57) and (3.58) of [2]. In (9.13)–(9.17), (9.24), and (9.25),

$$
\mathrm{N} = n + 1 \quad \text{and} \quad \mathrm{N} = 1, 2, \cdots, \mathrm{NMAX}. \tag{9.27}
$$

In (9.13) and (9.14), $n^{\delta +}c$ and $n^{\delta -}c$ are given, respectively, by eqs. (3.79) and (3.78) of [2] with q replaced with n. In (9.15), $\hat{D}_n^{\delta}$ is given by eq. (3.82) of [2]. In (9.16), $\hat{G}_n^{\delta}$ is given by eq. (3.80) of [2] with q replaced with n. The right-hand side of (9.17) appears in eq. (3.90) of [2]. In (9.20)–(9.23), z_{ee}, z_o, z_{oe}, and z_{oo} are given, respectively, by eqs. (3.99)–(3.102) of [2]. In (9.24), $\hat{D}_n^{(3)}$ is given by eq. (3.111) of [2]. In (9.25), $\hat{G}_n^{(4)}$ is given by eq. (3.109) of [2] with q replaced with n.

Not all output variables are always calculated. The variables CP(N), CM(N), D(N), and G(N) are calculated only when ICUT=1. The variables D3(N) and G4(N) are calculated only when ICUT=1 and ITMTE=2. The variables DQ(N), GCS, GC2, ZEE, ZZ, ZOE, ZOO, and PGC are calculated only if ICUT=2.

9.3 Description of the Subroutine DGN

The seventh statement before DO loop 12 sets

$$\text{GAM2} = \text{XX} - (ka)^2 \tag{9.28}$$

where XX is given by (9.10). If

$$(ka)^2 > \text{XX}, \tag{9.29}$$

then the next statement, which is

IF(GAM2.GE.0.D+0) GO TO 11,

allows execution to continue on to DO loop 12 and eventually to the return statement immediately thereafter. If

$$(ka)^2 \leq \text{XX}, \tag{9.30}$$

then control passes to statement 11 and eventually to the return statement immediately after statement 20.

9.3.1 Above Cutoff

In this section, the wavenumber k is above the cutoff wavenumber $\sqrt{\text{XX}}/a$ so that (9.29) holds. As a result, DO loop 12 and the five statements before it are executed. The fifth, fourth, third, and second statements before DO loop 12 set

$$\text{ICUT} = 1 \tag{9.31}$$

$$\text{GAM} = \beta^{\delta}_{rs} a \tag{9.32}$$

$$\text{BL} = \beta^{\delta}_{rs} L_3 \tag{9.33}$$

$$\text{BC5} = \frac{\beta^{\delta}_{rs} c}{2}. \tag{9.34}$$

In DO loop 12,

$$\text{N} = n + 1 \tag{9.35}$$

where n is the integer that appears in (9.13)–(9.16), (9.24), and (9.25). The first four statements in DO loop 12 set

$$\mathrm{SGN} = (-1)^n \tag{9.36}$$

$$\mathrm{PIN} = \frac{n\pi}{2} \tag{9.37}$$

$$\mathrm{CNP} = \frac{n^{\delta+}c}{2} \tag{9.38}$$

$$\mathrm{CNM} = \frac{n^{\delta-}c}{2} \tag{9.39}$$

where $n^{\delta+}c$ and $n^{\delta-}c$ are given, respectively, by eqs. (3.79) and (3.78) of [2] with q replaced with n. The next two statements in DO loop 12 set CP(N) and CM(N) equal to the right-hand sides of (9.13) and (9.14), respectively. Statement 21 and the three statements prior to it set

$$\mathrm{SP} = \frac{\sin\left(\dfrac{n^{\delta+}c}{2}\right)}{\left(\dfrac{n^{\delta+}c}{2}\right)} \tag{9.40}$$

where it is understood that when $n^{\delta+}c = 0$, SP is to be replaced by its limit as $n^{\delta+}c$ approaches zero. This limit is given by

$$\lim_{n^{\delta+}c \to 0} \mathrm{SP} = 1. \tag{9.41}$$

Statement 14 and the three statements prior to it set

$$\mathrm{SM} = \frac{\sin\left(\dfrac{n^{\delta-}c}{2}\right)}{\left(\dfrac{n^{\delta-}c}{2}\right)} \tag{9.42}$$

where it is understood that the right-hand side of (9.42) is to be replaced with 1 when $n^{\delta-}c = 0$.

Statement 15 and the seven statements following it set

$$\mathrm{ARG} = \beta^{\delta}_{rs} L_3 - \frac{n\pi}{2} \tag{9.43}$$

$$\mathrm{CS} = \cos\left(\beta^{\delta}_{rs} L_3 - \frac{n\pi}{2}\right) \tag{9.44}$$
$$\mathrm{SN} = \sin\left(\beta^{\delta}_{rs} L_3 - \frac{n\pi}{2}\right) \tag{9.45}$$
$$\mathrm{C1} = (-1)^n \frac{\sin\left(\dfrac{n^{\delta-}c}{2}\right)}{\left(\dfrac{n^{\delta-}c}{2}\right)} \tag{9.46}$$
$$\mathrm{C2} = \frac{\sin\left(\dfrac{n^{\delta+}c}{2}\right)}{\left(\dfrac{n^{\delta+}c}{2}\right)} + (-1)^n \frac{\sin\left(\dfrac{n^{\delta-}c}{2}\right)}{\left(\dfrac{n^{\delta-}c}{2}\right)} \tag{9.47}$$
$$\mathrm{U1} = -\sin\left(\beta^{\delta}_{rs} L_3 - \frac{n\pi}{2}\right) - j\cos\left(\beta^{\delta}_{rs} L_3 - \frac{n\pi}{2}\right) \tag{9.48}$$
$$\mathrm{D(N)} = \hat{D}^{\delta}_{n} \tag{9.49}$$
$$\mathrm{G(N)} = \hat{G}^{\delta}_{n}. \tag{9.50}$$

In (9.49), $\hat{D}^{\delta}_{n}$ is given by eq. (3.82) of [2]. In (9.50), $\hat{G}^{\delta}_{n}$ is given by eq. (3.80) of [2] with q replaced by n.

The statement

GO TO (12,13), ITMTE

sends execution directly to statement 12 if ITMTE = 1. If ITMTE = 2, then statement 13 and the two statements following it set

$$\mathrm{C3} = \frac{\sin\left(\dfrac{n^{\delta+}c}{2}\right)}{\left(\dfrac{n^{\delta+}c}{2}\right)} - (-1)^n \frac{\sin\left(\dfrac{n^{\delta-}c}{2}\right)}{\left(\dfrac{n^{\delta-}c}{2}\right)} \tag{9.51}$$
$$\mathrm{D3(N)} = \hat{D}^{(3)}_{n} \tag{9.52}$$
$$\mathrm{G4(N)} = \hat{G}^{(4)}_{n}. \tag{9.53}$$

In (9.52), $\hat{D}^{(3)}_{n}$ is given by eq. (3.111) of [2] with TE replaced by δ. In (9.53), $\hat{G}^{(4)}_{n}$ is given by eq. (3.109) of [2] with q and TE replaced, respectively, by n and δ.

9.3.2 Below or at Cutoff

In this section, the wavenumber k is below or at the cutoff wavenumber $\sqrt{\mathrm{XX}}/a$ so that (9.30) holds. As a result, control passes from the sixth statement before DO loop 12 to statement 11 and eventually to DO loop 20.

Statement 11 and the thirteen statements following it set

$$\mathrm{ICUT} = 2 \tag{9.54}$$
$$\mathrm{GAM} = ga \tag{9.55}$$
$$\mathrm{GC} = gc \tag{9.56}$$
$$\mathrm{GCS} = (gc)^2 \tag{9.57}$$
$$\mathrm{GC2} = 2gc \tag{9.58}$$
$$\mathrm{PGC} = \frac{\pi}{gc} \tag{9.59}$$
$$\mathrm{GL2} = 2gL_3 \tag{9.60}$$
$$\mathrm{EL} = e^{-2gL_3} \tag{9.61}$$
$$\mathrm{EC} = e^{-gc} \tag{9.62}$$
$$\mathrm{ELCM} = e^{-2gL_3+gc} \tag{9.63}$$
$$\mathrm{ELCP} = e^{-2gL_3-gc} \tag{9.64}$$
$$\mathrm{EE} = 2\left(e^{-2gL_3+gc} + e^{-2gL_3-gc}\right) \tag{9.65}$$
$$\mathrm{ECEL} = e^{-gc} - e^{-2gL_3} \tag{9.66}$$
$$\mathrm{GCX4} = 4gc \tag{9.67}$$

where, as in eq. (3.89) of [2],

$$g = \gamma^{\delta}_{rs}\,. \tag{9.68}$$

If $gc < 1$, control passes to the statement after the branch statement

IF(GC.GE.1) GO TO 16

and eventually to statement 18. The effects of the statement after the above-mentioned branch statement and all further statements up to and including statement 18 are described in this paragraph. The five statements after the branch statement

IF(GC.GE.1) GO TO 16

set

$$
\begin{aligned}
\text{GC5} &= \frac{gc}{2} && (9.69)\\
\text{EC5} &= e^{-\frac{gc}{2}} && (9.70)\\
\text{SC5} &= \sinh\left(\frac{gc}{2}\right) && (9.71)\\
\text{ELSC5} &= e^{-2gL_3}\sinh\left(\frac{gc}{2}\right) && (9.72)\\
\text{ZEE} &= -4z_{ee} \ \text{ for } \ gc < 1\,. && (9.73)
\end{aligned}
$$

In (9.73), z_{ee} is given by the second right-hand side of eq. (3.99) of [2]. The statement after the branch statement

IF(GC.GE.0.01D+0) GO TO 17

and all further statements up to and including statement 23 set

$$\text{ZZ} = -4z_o \ \text{ for } \ gc < 0.01. \tag{9.74}$$

The above z_o is given by the third right-hand side of eq. (3.100) of [2] with the $(gc/2)^5$ and $(gc/2)^7$ terms omitted when $(gc/2) < 10^{-4}$. These terms were omitted when $(gc/2) < 10^{-4}$ because they are very small compared to the $(gc/2)^3$ term when $(gc/2) < 10^{-4}$. Statement 17 sets

$$\text{ZZ} = -4z_o \ \text{ for } \ 0.01 \le gc < 1 \tag{9.75}$$

where z_o is given by the second right-hand side of eq. (3.100) of [2]. Statement 18 sets

$$\text{ZOE} = -4z_{oe} \ \text{ for } \ gc < 1 \tag{9.76}$$

where z_{oe} is given by the second right-hand side of eq. (3.101) of [2].

If $gc \ge 1$, the branch statement

IF(GC.GE.1) GO TO 16

sends execution to statement 16. Statement 16 and two statements following it set

$$
\begin{aligned}
\text{ZEE} &= -4z_{ee} \ \text{ for } \ gc \ge 1 && (9.77)\\
\text{ZZ} &= -4z_o \ \text{ for } \ gc \ge 1 && (9.78)\\
\text{ZOE} &= -4z_{oe} \ \text{ for } \ gc \ge 1 && (9.79)
\end{aligned}
$$

where z_{ee}, z_o, and z_{oe} are given, respectively, by the first right-hand sides of eqs. (3.99)–(3.101) of [2].

Statement 19 sets

$$\mathrm{ZOO} = -4z_{oo} \tag{9.80}$$

where z_{oo} is given by the first right-hand side of eq. (3.102) of [2]. The second right-hand side of eq. (3.102) of [2] was never used because it was deemed nearly as susceptible to roundoff error as the first right-hand side of eq. (3.102) of [2] when $gc < 1$. The second statement in DO loop 20 sets

$$\mathrm{DQ(N)} = \frac{1}{(n\pi)^2 + (gc)^2} \tag{9.81}$$

where N and n are related by

$$\mathrm{N} = n + 1. \tag{9.82}$$

9.4 Listing of the Subroutine DGN

```
      SUBROUTINE DGN(ITMTE,X,XX,ICUT,GAM,CP,CM,D,G,DQ,GCS,GC2,ZEE,
     1ZZ,ZOE,ZOO)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /DGN/S,BKA2,L3,C,C5,PI5,D3(50),G4(50),PGC
      COMMON /PI/PI
      COMMON /NMAX/NMAX
      COMPLEX*16 U1,D(50),D3
      REAL*8 X(200),CP(50),CM(50),DQ(50),G(50),L3
      INTEGER S
      XX=X(S)
      XX=XX*XX
      GAM2=XX-BKA2
      IF(GAM2.GE.0.D+0) GO TO 11
      ICUT=1
      GAM=DSQRT(-GAM2)
      BL=GAM*L3
      BC5=GAM*C5
      SGN=-1.D+0
      DO 12 N=1,NMAX
      SGN=-SGN
      PIN=PI5*(N-1)
```

```
      CNP=PIN+BC5
      CNM=PIN-BC5
      CP(N)=CNP*2.D+0
      CM(N)=CNM*2.D+0
      IF(CNP.NE.0.D+0) GO TO 21
      SP=1.D+0
      GO TO 22
   21 SP=DSIN(CNP)/CNP
   22 IF(CNM.NE.0.D+0) GO TO 14
      SM=1.D+0
      GO TO 15
   14 SM=DSIN(CNM)/CNM
   15 ARG=BL-PIN
      CS=DCOS(ARG)
      SN=DSIN(ARG)
      C1=SGN*SM
      C2=SP+C1
      U1=-DCMPLX(SN,CS)
      D(N)=C2*U1
      G(N)=C2*CS
      GO TO (12,13), ITMTE
   13 C3=SP-C1
      D3(N)=C3*U1
      G4(N)=-C3*CS
   12 CONTINUE
      RETURN
   11 ICUT=2
      GAM=DSQRT(GAM2)
      GC=GAM*C
      GCS=GC*GC
      GC2=2.D+0*GC
      PGC=PI/GC
      GL2=GAM*L3*2.D+0
      EL=DEXP(-GL2)
      EC=DEXP(-GC)
      ELCM=DEXP(-GL2+GC)
      ELCP=DEXP(-GL2-GC)
      EE=(ELCM+ELCP)*2.D+0
      ECEL=EC-EL
      GCX4=GC*4.D+0
      IF(GC.GE.1) GO TO 16
      GC5=GAM*C5
      EC5=DEXP(-GC5)
      SC5=DSINH(GC5)
```

```
      ELSC5=EL*SC5
      ZEE=8.D+0*(EC5-ELSC5)*SC5
      IF(GC.GE.0.01D+0) GO TO 17
      G2=GC5*GC5
      G3=G2*GC5
      GC4=GC/4.D+0
      ZZ=G3/6.D+0-(2.D+0*DEXP(-GC4)*DSINH(GC4)+ELSC5)*SC5
      IF(GC5.LT.1.D-4) GO TO 23
      G5=G2*G3
      ZZ=ZZ+G5/120.D+0+G2*G5/5040.D+0
   23 ZZ=8.D+0*ZZ
      GO TO 18
   17 ZZ=8.*(EC5-ELSC5)*SC5-GCX4
   18 ZOE=4.D+0*EL*DSINH(GC)
      GO TO 19
   16 ZEE=4.*(1.-ECEL)-EE
      ZZ=ZEE-GCX4
      ZOE=(ELCM-ELCP)*2.D+0
   19 ZOO=4.*(1.+ECEL)-EE
      DO 20 N=1,NMAX
      PIN=PI*(N-1)
      DQ(N)=1.D+0/(PIN*PIN+GCS)
   20 CONTINUE
      RETURN
      END
```

Chapter 10

The Function Subprogram FXY

The function subprogram FXY sets

$$\mathrm{FXY(I, X, Y,)} = f(x, y) \tag{10.1}$$

where

$$\mathrm{X} = x \tag{10.2}$$
$$\mathrm{Y} = y. \tag{10.3}$$

Furthermore, x and y are such that

$$x + y = I\pi \tag{10.4}$$

and $f(x, y)$ is given by eq. (3.84) of [2].

10.1 Description of the Function Subprogram FXY

In the listing of the function subprogram FXY in Section 10.2, there are six statements that define FXY. If $x + y = 0$, the first and second of these statements set[†]

$$\mathrm{FXY} = \begin{cases} \dfrac{y}{3!}, & |y| < 10^{-5} \\ \dfrac{y}{3!} - \dfrac{y^3}{5!} + \dfrac{y^5}{7!}, & 10^{-5} \leq |y| \leq 0.1 \end{cases} \tag{10.5}$$

and the third one sets

$$\mathrm{FXY} = \frac{y - \sin y}{y^2}, \quad |y| > 0.1. \tag{10.6}$$

In (10.5), the y^3 and y^5 terms are absent when $|y| < 10^{-5}$. These terms were omitted because their magnitudes are very small compared to $|y|/3!$ when $|y| < 10^{-5}$. If $x + y \neq 0$, the fourth and fifth of these statements set

$$\mathrm{FXY} = \frac{(-1)^I \sin y}{yx}, \quad |y| \leq 1.57 \tag{10.7}$$

and the sixth one sets

$$\mathrm{FXY} = -\frac{\sin x}{yx}, \quad |y| > 1.57. \tag{10.8}$$

10.2 Listing of the Function Subprogram FXY

```
      FUNCTION FXY(I,X,Y)
      IMPLICIT REAL*8 (A-H,O-Z)
      YA=DABS(Y)
      IF(I.NE.0) GO TO 11
      IF(YA.GT..1D+0) GO TO 12
      FXY=Y/6.D+0
      IF(YA.LT.1.D-5) RETURN
      Y2=Y*Y
      Y3=Y2*Y
      FXY=FXY-Y3/120.D+0+Y3*Y2/5040.D+0
      RETURN
   12 FXY=(Y-DSIN(Y))/(Y*Y)
      RETURN
```

[†]See eq. (3.84) of [2].

```
11 IF(YA.GT.1.57D+0) GO TO 13
   FXY=DSIN(Y)/(Y*X)
   IF((I-2*(I/2)).NE.0) FXY=-FXY
   RETURN
13 FXY=-DSIN(X)/(Y*X)
   RETURN
   END
```

Chapter 11

The Subroutines DECOMP and SOLVE

The subroutines DECOMP and SOLVE solve the linear equation system

$$A\vec{x} = \vec{b} \tag{11.1}$$

where A is an $n \times n$ matrix, $\vec{x}$ is an $n \times 1$ column vector of n unknowns, and $\vec{b}$ is an $n \times 1$ column vector of of n knowns.

11.1 Input and Output Data and Minimum Allocations

The input to the subroutine DECOMP(N,IPS,UL) consists of $N = n$ and the n^2 elements of the matrix A stored by columns in the one-dimensional array UL. The output from the subroutine DECOMP is IPS and UL.[†] This output is fed into the subroutine SOLVE(N,IPS,UL,B,X). The rest of the input to the subroutine SOLVE consists of N and the n elements of $\vec{b}$ stored in the one-dimensional array B. The subroutine SOLVE puts the n elements of $\vec{x}$ in the one-dimensional array X. The linear equation system (11.1) can be solved for several different column vectors $\vec{b}$ by calling the subroutine

[†] The subroutine DECOMP calculates n entries of IPS and changes the n^2 entries of UL.

DECOMP once and calling the subroutine SOLVE several times, once for each $\vec{b}$.

Minimum allocations are given by

```
COMPLEX UL(N*N)
DIMENSION SCL(N),IPS(N)
```

in the the subroutine DECOMP and by

```
COMPLEX UL(N*N),B(N),X(N)
DIMENSION IPS(N)
```

in the subroutine SOLVE.

The functioning of the subroutines DECOMP and SOLVE is described on pages 46–49 of [7].

11.2 Listing of the Subroutines DECOMP and SOLVE

```
  SUBROUTINE DECOMP (N,IPS,UL)
  COMPLEX UL(24336),PIVOT,EM
  DIMENSION SCL(156),IPS(156)
  DO 5 I=1,N
  IPS(I)=I
  RN=0.
  J1=I
  DO 2 J=1,N
  ULM=ABS(REAL(UL(J1)))+ABS(AIMAG(UL(J1)))
  J1=J1+N
  IF(RN-ULM) 1,2,2
1 RN=ULM
2 CONTINUE
  SCL(I)=1./RN
5 CONTINUE
  NM1=N-1
  K2=0
  DO 17 K=1,NM1
  BIG=0.
  DO 11 I=K,N
  IP=IPS(I)
  IPK=IP+K2
  SIZE=(ABS(REAL(UL(IPK)))+ABS(AIMAG(UL(IPK))))*SCL(IP)
```

```
      IF(SIZE-BIG) 11,11,10
   10 BIG=SIZE
      IPV=I
   11 CONTINUE
      IF(IPV-K) 14,15,14
   14 J=IPS(K)
      IPS(K)=IPS(IPV)
      IPS(IPV)=J
   15 KPP=IPS(K)+K2
      PIVOT=UL(KPP)
      KP1=K+1
      DO 16 I=KP1,N
      KP=KPP
      IP=IPS(I)+K2
      EM=-UL(IP)/PIVOT
   18 UL(IP)=-EM
      DO 16 J=KP1,N
      IP=IP+N
      KP=KP+N
      UL(IP)=UL(IP)+EM*UL(KP)
   16 CONTINUE
      K2=K2+N
   17 CONTINUE
      RETURN
      END
      SUBROUTINE SOLVE(N,IPS,UL,B,X)
      COMPLEX UL(24336),B(156),X(156),SUM
      DIMENSION IPS(156)
      NP1=N+1
      IP=IPS(1)
      X(1)=B(IP)
      DO 2 I=2,N
      IP=IPS(I)
      IPB=IP
      IM1=I-1
      SUM=0.
      DO 1 J=1,IM1
      SUM=SUM+UL(IP)*X(J)
    1 IP=IP+N
    2 X(I)=B(IPB)-SUM
      K2=N*(N-1)
      IP=IPS(N)+K2
      X(N)=X(N)/UL(IP)
      DO 4 IBACK=2,N
```

```
      I=NP1-IBACK
      K2=K2-N
      IPI=IPS(I)+K2
      IP1=I+1
      SUM=0.
      IP=IPI
      DO 3 J=IP1,N
      IP=IP+N
    3 SUM=SUM+UL(IP)*X(J)
    4 X(I)=(X(I)-SUM)/UL(IPI)
      RETURN
      END
```

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